Thermalization in quantum systems III: Integrable models

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Motivation

- Thermalization in closed quantum systems, Statistical physics?
- Exceptions: Integrable models
- Gibbs Ensemble, Generalized Gibbs Ensemble
- Exact solutions in integrable models

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Setting the stage

Equilibration after global quenches

- The models: 1D spin chains given by a local Hamiltonian H
- Initial state: $|\Psi_0\rangle$
  - Ground state of a local Hamiltonian H_0
  - States prepared according to simple (local) rules
    Examples:
    $$|\Psi_0\rangle = |N\rangle = \otimes_{k=1}^{L/2} |\uparrow\downarrow\rangle$$
    or
    $$|\Psi_0\rangle = |D\rangle = \otimes_{k=1}^{L/2} \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$
  - Satisfies the cluster decomposition principle for local operators:
    $$\lim_{x\to\infty} \langle\Psi_0|\mathcal{O}(y)\mathcal{O}(x+y)|\Psi_0\rangle = \langle\Psi_0|\mathcal{O}|\Psi_0\rangle^2$$
- Question: If $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$, then for $\mathcal{O}$ local operators
  $$\lim_{t\to\infty} \langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = ???$$

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Thermalization

Time dependence of local observables:

$$\langle \mathcal{O}(t) \rangle = \sum_{n,m} \langle \Psi_0 | n \rangle \langle n | \mathcal{O} | m \rangle \langle m | \Psi_0 \rangle e^{-it(E_m - E_n)}$$

Long-time limit, diagonal ensemble:

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t dt' \, \langle \mathcal{O}(t') \rangle = \sum_n |c_n|^2 \langle n | \mathcal{O} | n \rangle, \qquad c_n = \langle \Psi_0 | n \rangle$$

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Find T from the initial state and make predictions!

Only one T for all local observables!

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Eigenstate Thermalisation Hypothesis (ETH): $\langle\Psi|\mathcal{O}|\Psi\rangle = f(E/L)$

$$\text{DE} = \text{GE}$$

$$\sum_n |c_n|^2 \langle n|\mathcal{O}|n\rangle \approx \text{Tr}[\mathcal{O}\rho_G]$$

Unrelated weights, but the same energy density!

CDP:

$$\frac{\Delta E}{L} = \frac{\sqrt{\langle\Psi_0|H^2|\Psi_0\rangle - \langle\Psi_0|H|\Psi_0\rangle^2}}{L} \sim \frac{1}{\sqrt{L}}$$

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Integrable models

Definition?

- Exactly solvable
- Bethe Ansatz solvable (XXX Heisenberg spin chain)
- Existence of a set of higher charges

XXZ Hamiltonian:

$$H = \sum_{j=1}^{L} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta(\sigma_j^z \sigma_{j+1}^z - 1))$$

Example for a new charge: (at $\Delta = 1$)

$$Q_3 = \sum_{j=1}^{L} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\sigma}_{j+1} \times \boldsymbol{\sigma}_{j+2})$$

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Quench from the Néel state, $\Delta = 3$

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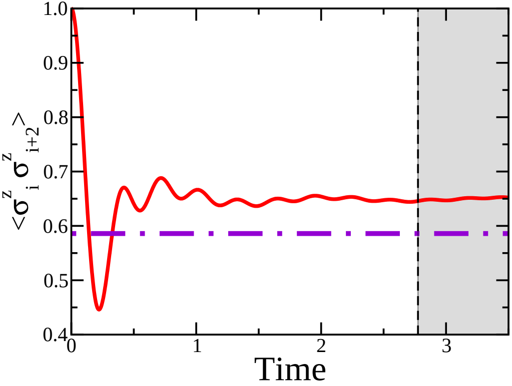

iTEBD simulation, Miklós Werner

GGE

Higher conserved charges: $\{Q_j\}$, such that

$$[Q_j, Q_k] = 0$$

They are extensive: $|Q_j| \sim L$

$H \in \{Q_j\}$

In a finite chain: number of operators grows polynomially with L.

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$$\langle \mathcal{O} \rangle_{GGE} = \mathrm{Tr}[\mathcal{O}\rho_{GGE}] \qquad \rho_{GGE} = \frac{e^{-\sum_j \lambda_j Q_j}}{\mathrm{Tr}\ e^{-\lambda_j Q_j}}$$

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Lagrange multipliers are fixed from

$$\langle \Psi_0|Q_j|\Psi_0\rangle = \mathrm{Tr}[Q_j\ \rho_{GGE}] \qquad j = 1\ldots\infty$$

Find them and make predictions!

Problems: Infinite number of operators, norm, locality, etc.

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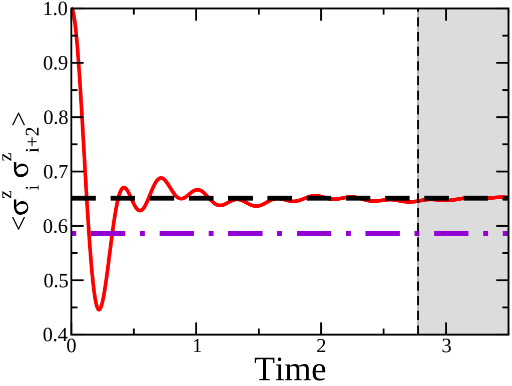

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XX model

$$H = \sum_{j=1}^{L} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y)$$

Solution: Jordan Wigner transformation.

$$c_j^\dagger = e^{i\pi \sum_{k=1}^{j-1} \sigma_k^+ \sigma_k^-} \sigma_j^+ \qquad c_j = e^{-i\pi \sum_{k=1}^{j-1} \sigma_k^+ \sigma_k^-} \sigma_j^-$$

We get the fermionic relations:

$$\{c_j, c_k\} = \{c_j^\dagger, c_k^\dagger\} = 0, \qquad \{c_j^\dagger, c_k\} = \delta_{j,k}$$

Hamiltonian is written as $H = \sum_k \varepsilon_k \tilde{c}_k^\dagger \tilde{c}_k$, where

$$\tilde{c}_k^\dagger = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{ikj} c_j^\dagger \qquad \tilde{c}_k = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{-ikj} c_j$$

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Charges: $\tilde{Q}_k = \tilde{n}_k = \tilde{c}_k^\dagger \tilde{c}_k$
GGE:

$$\rho_{GGE} = \frac{1}{Z} e^{-\sum_k \tilde{\lambda}_k \tilde{n}_k}$$

States in the TDL: Given by density $n(k)$.

GETH: In the TDL the mean values of local operators can be expressed using $n(k)$ only (Wick theorem).

Example:

$$\langle N_1 N_2 \rangle = \int \frac{dk_1}{2\pi} \int \frac{dk_2}{2\pi} \; n(k_1) n(k_2) (1 + \cos(k_1 - k_2))$$

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What are the properties of this GGE? $\rho_{GGE} = \frac{1}{Z} e^{-\sum_k \tilde{\lambda}_k \tilde{n}_k}$

- In the thermal case we would have $\tilde{\lambda}_k = \frac{e_k}{T}$
- Mode dependent temperatures (completely fixed by the initial state)
- Measured by experiment! *Experimental observation of a generalized Gibbs ensemble*, T. Langen et. al., Science 348 (2015) 207-211
- Highly non-local!
- Number of parameters grows linearly with the volume: still predictive!

Partial solution to the locality problem: New basis for the charges.

$$Q_j = \sum_k 2\cos(jk)\tilde{n}_k = \sum_{l=1}^{L} (c_l^\dagger c_{l+j} + cc.)$$

A local GGE?

$$\rho_{GGE} = \frac{1}{Z} e^{-\sum_j \lambda_j Q_j}$$

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- In the thermal case we would have $\tilde{\lambda}_k = \frac{\varepsilon_k}{T}$
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Truncated GGE

$$\rho^{(n)}_{GGE} = \frac{1}{Z} e^{-\sum_j^n \lambda_j Q_j}$$

For local observables the limit exists:

$$\langle \mathcal{O} \rangle_{GGE} = \lim_{n\to\infty} \mathrm{Tr}\left[\rho^{(n)}_{GGE} \mathcal{O}\right]$$

At every n the ensemble is local, and

$$\lim_{t\to\infty} \langle \mathcal{O}(t) \rangle = \lim_{n\to\infty} \mathrm{Tr}\left[\rho^{(n)}_{GGE} \mathcal{O}\right]$$

Role of locality: most local observables converge most quickly

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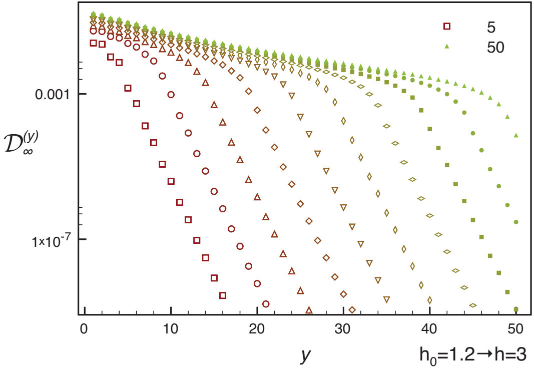

$$\left| \mathrm{Tr}_{L-l}(\rho_{GGE}) - \mathrm{Tr}_{L-l}(\rho^{(y)}_{GGE}) \right|$$

Interacting XXZ model

$$H = \sum_{j=1}^{L} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \cosh(\eta)(\sigma_j^z \sigma_{j+1}^z - 1))$$

- Solvable by the Bethe Ansatz: two-particle reducible scattering

$$e^{ip} = \frac{\sinh(\lambda + i\eta/2)}{\sinh(\lambda - i\eta/2)} \qquad S(\lambda_1, \lambda_2) = \frac{\sinh(\lambda_1 - \lambda_2 - i\eta)}{\sinh(\lambda_1 - \lambda_2 + i\eta)}$$

- Spin waves can form bound states: so-called strings

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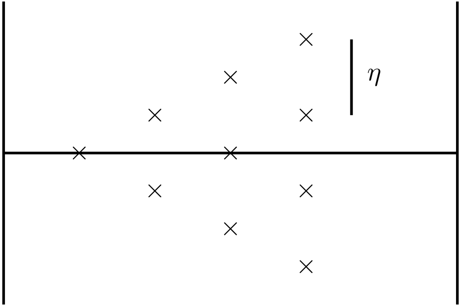

XXZ chain

- Physical picture: bound states – different particles
- In the TDL: densities for the k-strings $\rho_k(\lambda)$
- GETH: $\langle\Psi|\mathcal{O}|\Psi\rangle = \mathcal{O}(\{\rho_k(p)\})$

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XXZ chain

Quasi-local, quasi-conserved operators $Q_{s,j}$ with $s, j = 1 \dots \infty$
Operator norm: $|Q_{s,j}|^2 \sim L$

$$Q_{2,1} \sim \sum_{j=1}^{L} \Big[S_j \cdot S_{j+2} + \frac{155}{252} S_j \cdot S_{j+3} + \frac{64}{63} (S_j \cdot S_{j+1})(S_{j+2} \cdot S_{j+3}) - \\ - \frac{212}{84} (S_j \cdot S_{j+2})(S_{j+1} \cdot S_{j+3}) - \frac{44}{84} (S_j \cdot S_{j+3})(S_{j+1} \cdot S_{j+2}) \Big] + \dots$$

These charges are important. Without them the GGE does not give good predictions.

B. Wouters et. al., Phys. Rev. Lett. 113 (2014), 117202
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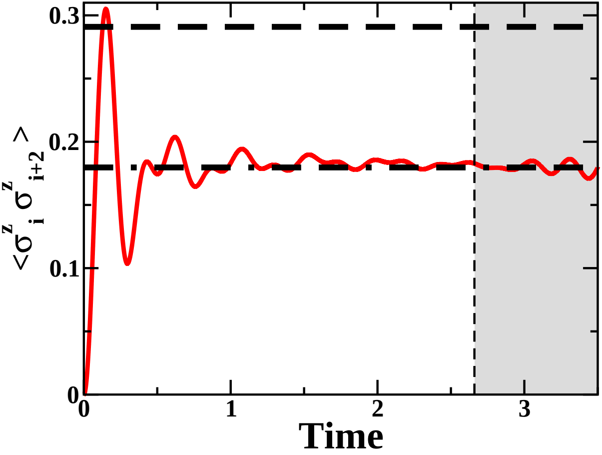
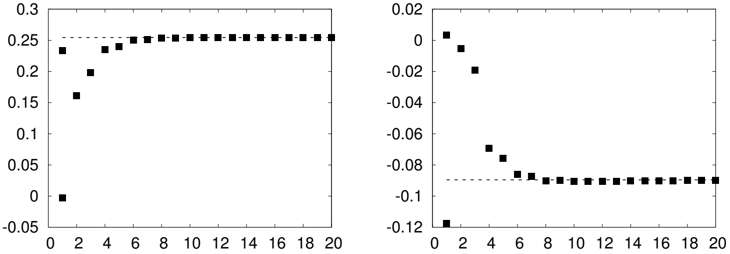
Quench from the Dimer state, $\Delta = 3$

$$|\Psi(t=0)\rangle = \otimes_{j=1}^{L/2} \frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{\sqrt{2}}$$



Truncated GGE

The correlators $\langle \sigma_1^z \sigma_3^z \rangle$ and $\langle \sigma_1^z \sigma_4^z \rangle$ after adding n charges from the first n families



Thank you for your attention!

$$H_{XXZ} = \sum_{j=1}^{L} \left\{ \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta(\sigma_j^z \sigma_{j+1}^z - 1) \right\}$$

Bethe Ansatz equations for $\Delta = \cosh(\eta) > 1$:

$$e^{ip_j L} = \left(\frac{\sin(\lambda_j + i\eta/2)}{\sin(\lambda_j - i\eta/2)} \right)^L = \prod_{k \neq j} \frac{\sin(\lambda_j - \lambda_k + i\eta)}{\sin(\lambda_j - \lambda_k - i\eta)}$$

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String solutions:

Densities of roots: $\rho_{\mathsf{r},k}(\lambda)$

The number ΔN of k-strings with centers between λ and $\lambda + \Delta\lambda$: $\Delta N = L\rho_{\mathsf{r},k}(\lambda)\Delta\lambda/2\pi$.

Densities of holes: $\rho_{\mathsf{h},k}(\lambda)$.

They satisfy

$$\rho_{\mathsf{r},k} + \rho_{\mathsf{h},k} = \delta_{k,1} d + d \star (\rho_{\mathsf{h},k-1} + \rho_{\mathsf{h},k+1}) \, ,$$

where

$$(f \star g)(u) = \int_{-\pi/2}^{\pi/2} \frac{d\omega}{2\pi} f(u-\omega) g(\omega).$$

$$d(u) = 1 + 2 \sum_{n=1}^{\infty} \frac{\cos(2nu)}{\cosh(\eta n)}$$

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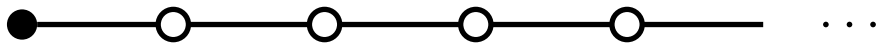

Generating function for the expectation values:

$$G_s(\lambda) = \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} \langle Q_{s,k} \rangle .$$

[*M. Fagotti and F. H. L. Essler, 2013*]

The following holds:

$$d \star (a_s + \rho_{h,s}) = G_s ,$$

[*B. Wouters et. al., 2015*]

Therefore:

$$|\Psi_0\rangle \quad \to \quad G_s(\lambda) \quad \to \quad \rho_{h,s}$$

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$$\eta_j(\lambda) \equiv \frac{\rho_{h,j}(\lambda)}{\rho_{r,j}(\lambda)} = e^{\varepsilon_j(\lambda)}$$

Exact solutions (at least for small j) have been derived in

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Y-system: $(\Delta = \cosh(\eta))$

$$\eta_j(\lambda + i\eta/2)\eta_j(\lambda_j - i\eta/2) = (1 + \eta_{j-1}(\lambda))(1 + \eta_{j+1}(\lambda))$$

- Holds for thermal states
- Was assumed for two-site product states
- Does NOT hold for quenches from certain 4-site product states, such as

$$|\Psi_0\rangle = \prod_{j=1}^{L/4} |\uparrow\uparrow\downarrow\downarrow\rangle$$

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- Allows for the computation of higher η_j efficiently

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$$\rho \sim \exp\left(\sum_s \int d\lambda\ \beta_s(\lambda) Q_s(\lambda)\right),$$

where

$$Q_s(\lambda) = \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} Q_{s,k}$$

It is derived:

$$\eta_j(\lambda + i\eta/2)\eta_j(\lambda_j - i\eta/2) = e^{\beta_j(\lambda)}(1+\eta_{j-1}(\lambda))(1+\eta_{j+1}(\lambda))$$

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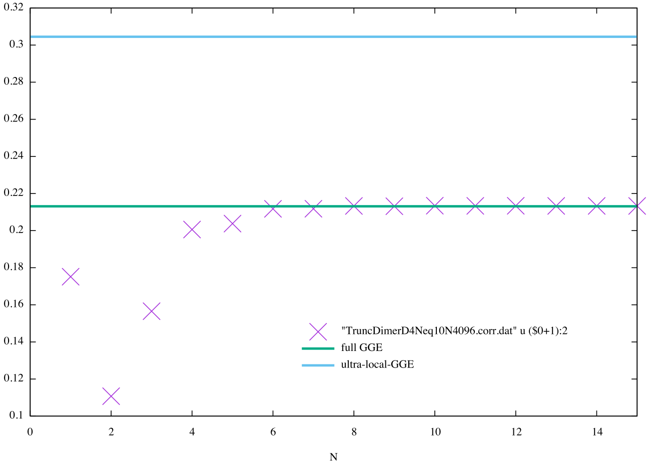
Solution: Truncated GGE!

There exists a series of tGGE density matrices ρ_N, $N = 1 \ldots \infty$ such that all local correlations evaluated using them tend to their physical values.

$$\rho_N \sim \exp\left(\sum_{s=1}^{N_s} \sum_{j=1}^{N_d} \beta_{s,j}^{(N)} Q_{s,j}\right)$$

(this is a theorem... more or less)

The $\langle\sigma_1^z\sigma_3^y\rangle$ correlator, Dimer quench, $\Delta = 4$, $\rho_N \sim \exp\left(\sum_{s,j=1}^{N} \beta_{s,j}^{(N)} Q_{s,j}\right)$



How does the proof work?

Generalized TBA for the density matrix $\rho_N \sim \exp\left(\sum_{s,j=1}^{N} \beta_{s,j}^N Q_{s,j}\right)$

$$\log \eta_j^N = \delta_{j \leq N} \sum_{k=1}^{N} \beta_{j,k}^N d^{(k)} + d \star (\log(1+\eta_{j-1}^N) + \log(1+\eta_{j+1}^N))$$

For the true η_j functions we can find the desired sources f_j from the integrals

$$\log \eta_j = f_j + d \star (\log(1+\eta_{j-1}) + \log(1+\eta_{j+1}))$$

We want: $\eta_j^N \to \eta_j$, therefore the source terms should match

$$\sum_{k=1}^{N} \beta_{j,k}^N d^{(k)} \to f_j$$

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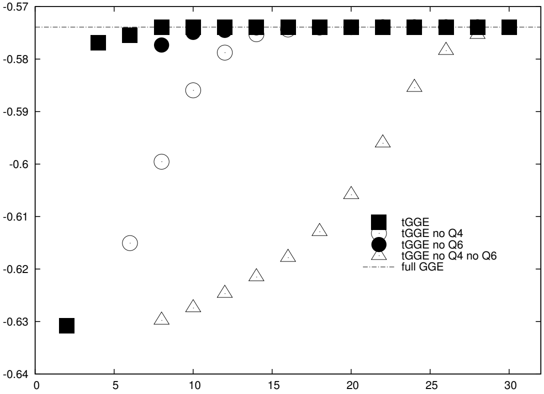
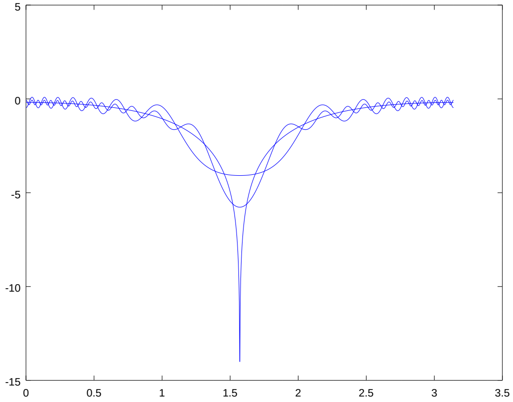
It works even if we leave out charges!



The Y-system relation is satisfied at each truncation:

$$\eta_j^N(\lambda + i\eta/2)\eta_j^N(\lambda_j - i\eta/2) = (1 + \eta_{j-1}^N(\lambda))(1 + \eta_{j+1}^N(\lambda))$$

Yet it can be broken for the limit $\eta_j = \lim_{N\to\infty} \eta_j^N$!


5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
3.5