Thermalization of Quantum Systems II.: Results from Quantum Information Theory

Zoltán Zimborás

Theoretical Physics Department, Wigner Research Centre for Physics

ELTE Statistical Physics Seminar
17 May 2017

- A central question about thermalization: Why is the Gibbs state special?

$$\rho_\beta = \frac{1}{Z} e^{-\beta H}, \quad Z = \mathrm{Tr}(e^{-\beta H}).$$

We discuss three points:

- 1st lecture (*Pure-state Statistical Physics*, Márton Kormos):
  It naturally emerges from subsystem expectmany-body dynamics.
- 2nd lecture (*Results from Quantum Information Theory*, ZZ):
  It naturally emerges when we introduce actors who can apply different unitary transformations from a given set.
- 3rd lecture (*Integrable Systems*, Balázs Pozsgay):
  Counter-examples, Generalized Gibbs Ensembles.

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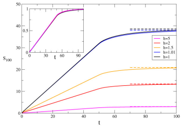
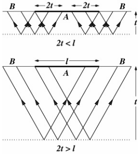
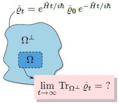
Last seminar talk: emergence of Gibbs state from unitary dynamics

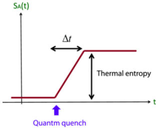
- Given a Hamiltonian H and a generic pure state with finite energy density. During the time-evolution the entanglement entropy increases linearly in time until $t \sim \ell$:





- The state on the local subsystems will be Gibbs states. The entanglement entropy will play the role of thermal entropy:





- The *Gibbs state* at *inverse temperature* β corresponding to a Hamiltonian H is given as

$$\rho_\beta = \frac{e^{-\beta H}}{\mathrm{Tr}(e^{-\beta H})}.$$

- We should define a *free energy functional* on the set of density matrices (corresponding inverse temperature β and Hamiltonian H)

$$F(\rho) = \mathrm{Tr}(\rho H) - \beta^{-1} S(\rho),$$

such that the state that maximizes this functional is exactly the Gibbs state ρ_β.

- We have to choose $S(\rho)$ to be the von Neumann entropy: $-\mathrm{Tr}\rho \log \rho$.

Gibbs states and the von Neumann entropy

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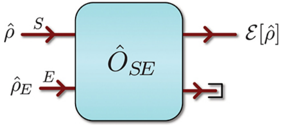
Quantum information approach: introducing actors who can use ancillary systems and unitaries

- The starting state $\rho_\beta = \rho_S \otimes \rho_E$ (or at least $\rho \approx \rho_S \otimes \rho_E$) that evolves unitarily.



- Why could $\rho = \rho_S \otimes \rho_E$ be justified? Doesn't that contradict our previous set-up (the entropy is built up from the entanglement with enviroment)?

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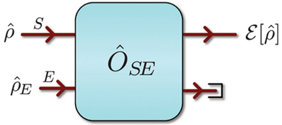

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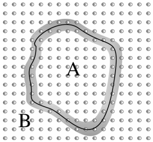
Subsystem are not very much entangled or correlated

- Monogamy of entanglement
- For **any short-ranged Gibbs** there is an area law for the mutual information of $I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ of neighboring regions and this mutual information will even decay exponentially with the distance (M.M. Wolf, F. Verstraete, M.B. Hastings, and J.I. Cirac, Phys. Rev. Lett. 100, 070502 (2008)).



- The above results also holds for other correlation measures:
  - Negativity V. Eisler, ZZ, New J. Phys. 16 , 123020 (2014); Phys. Rev. B 93, 115148 (2016).
  - Rényi mutual information: $D_i(\rho_{AB}||\rho_A \otimes \rho_B)$ where

$$D_1^{(\alpha)}(\rho\,||\,\sigma) = \frac{1}{\alpha - 1} \log \mathrm{Tr}\left(\rho^\alpha \sigma^{1-\alpha}\right) ,$$

$$D_2^{(\alpha)}(\rho\,||\,\sigma) = \frac{1}{\alpha - 1} \log \mathrm{Tr}\left(\sigma^{\frac{1-\alpha}{2\alpha}} \rho \sigma^{\frac{1-\alpha}{2\alpha}}\right)^\alpha .$$

(Martin Müller-Lennert et al, J. Math. Phys. 54, 122203 (2013); R. L. Frank, E. H. Lieb, J. Math. Phys. 54,122201 (2013).)

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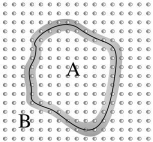

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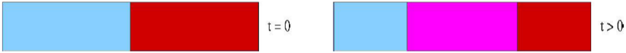
One counter-example: nonequilibrium steady state generated by a temperature gradient

- The initial state:

$$\rho_0 = \frac{1}{Z_L} e^{-\beta_\ell \mathcal{H}_\ell} \otimes \frac{1}{Z_R} e^{-\beta_r \mathcal{H}_r},$$

- Time evolution: $\rho(t) = e^{-itH} \rho_0 e^{itH}$, where

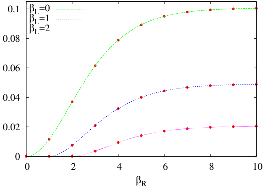
$$\mathcal{H} = -\frac{1}{2} \sum_{m=-\infty}^{\infty} \left(c_{m+1}^\dagger c_m + c_m^\dagger c_{m+1} \right)$$

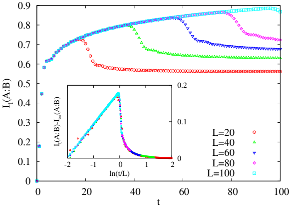


Violation of the area law

- Result: There is a logarithmic violation of the area law, $I(A:B) = \sigma \log L + k$,

$$\sigma = \frac{1}{\pi^2}\left[a\,\mathrm{Li}_2\left(\frac{a-b}{a}\right) + (1-a)\mathrm{Li}_2\left(\frac{b-a}{1-a}\right) + b\,\mathrm{Li}_2\left(\frac{b-a}{b}\right) + (1-b)\mathrm{Li}_2\left(\frac{a-b}{1-b}\right)\right],$$



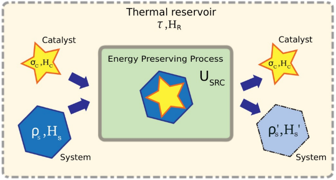


V. Eisler, Z.Zimborás, Phys. Rev. A 89, 032321 (2014),

M. Kormos, Z.Zimborás, arXiv:1612.04837 (2016).

- Similar counter examples appeared after ours:
M. Hoogeveen, B. Doyon, Nucl. Phys. B 898, 78-112 (2015), S. Ajisaka, F. Barra, B. Zunkovic, New J. Phys. 16 033028 (2014); F. Ares, J. G. Esteve, F. Falceto, E. Burillo, J. Phys. A 47, 245301 (2014).

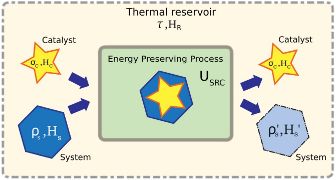
Catalytic thermal operations



$$\rho_S \otimes \tau^{\otimes n} \otimes \sigma \rightarrow U \rho_S \otimes \tau^{\otimes n} \otimes \sigma U^{\dagger},$$
$$[U, H_S + H_R + H_C] = 0.$$

- Borrow a catalyst that is returned to itself (or ϵ close to itself).
- Thermal reservoir: add arbitrary number of copies of a thermal state τ belonging to a Hamiltonian H_R.
- Perform any unitary operation U such that $[U, H_S + H_R + H_C]$.
- Perform the partial trace over the reservoir and the catalyst.

A Zeroth Law and Gibbs states



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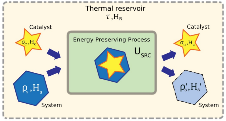
- A type of **Zeroth Law** of Thermodynamics:

  If we choose for τ any other form than $\frac{1}{Z} e^{-\beta H_R}$,
  then any $\rho_S \to \rho_S'$ transformation is possible.

  (F. Brandao, M. Horodecki, J. Oppenheim, J. Rennes and R.W. Spekkens, Phys. Rev. Lett. 111, 250404 (2013).)

A Second Law and the von Neumann entropy



- A type of **Second Law** of Thermodynamics:
  If in the above set-up a $\rho_S \to \rho'_S$ transformation is possible, then

$$F(\rho'_S) \leq F(\rho_S)$$

$$\mathrm{Tr}(H_S \rho'_S) - \beta^{-1} S(\rho'_S) \leq \mathrm{Tr}(H_S \rho_S) - \beta^{-1} S(\rho_S)$$

$$\beta^{-1} D(\rho'_S \| \rho_\beta) - \beta^{-1} \log Z \leq \beta^{-1} D(\rho_S \| \rho_\beta) - \beta^{-1} \log Z$$

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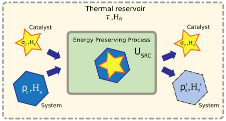

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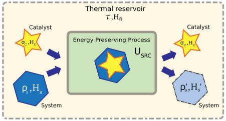

- However, the decrease of free energy is not the only restriction!
- **Second Laws of Thermodynamics**:
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It is then customary to introduce

$$F_i(\rho \| \rho_\beta) = D_i^{(\alpha)}(\rho'_S \| \rho_\beta) - \beta^{-1} \log Z.$$

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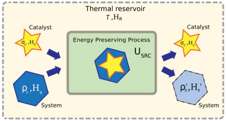

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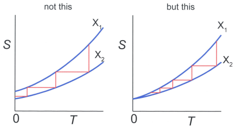
Extension of the Third Law of Thermodynamics



- Nernst's argument states that: at zero temperature, a finite size system has an entropy S, which is independent of any external parameters X, that is $S(T, X_1) - S(T, X_2) \to 0$ as the temperature $T \to 0$.
- As the entropy at zero temperature should be the logarithm of the ground state degeneracy, the validity of the third law is contingent on whether the degeneracy changes for different parameters.
- By modeling a physical process using two main ingredients:
  - finite information (entropy) propagation speed,
  - bound on the entropy $S(E) \leq \alpha V^{1-\nu} E^{\nu}$ with $\nu \in (1/2, \frac{d}{d+1})$,

one can derive a temperature-time bound

$$T_S \geq \frac{const.}{t^7}.$$

(L. Masanes and J. Oppenheim, Nature Comm. 8, 14538 (2017).)

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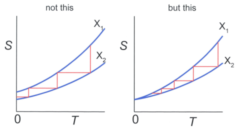

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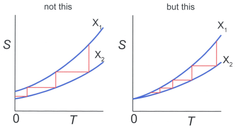

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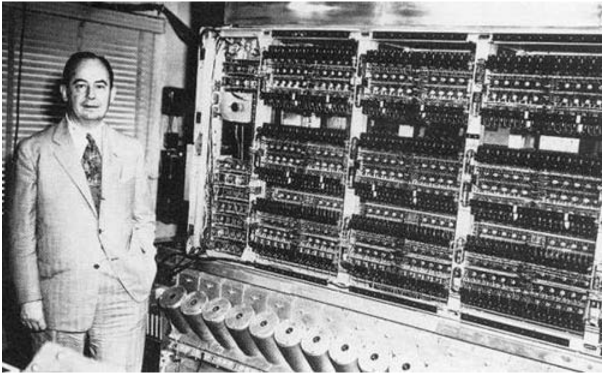
Quantum Computers with a Neumann Architecture



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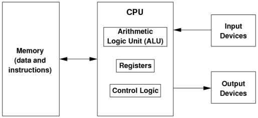
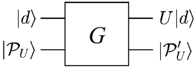


- The quantum mechanical version: Universally Programable Quantum Computer



- One needs a program input size that scales between $exp(d)$ and $exp(exp(d))$.

(M. A. Nielsen, I. L. Chuang, Phys. Rev. Lett., 79, 321 (1997).)

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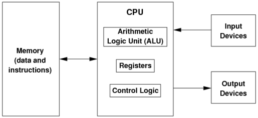

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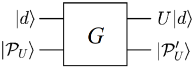

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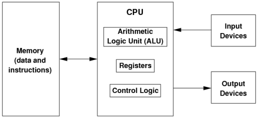

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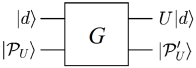

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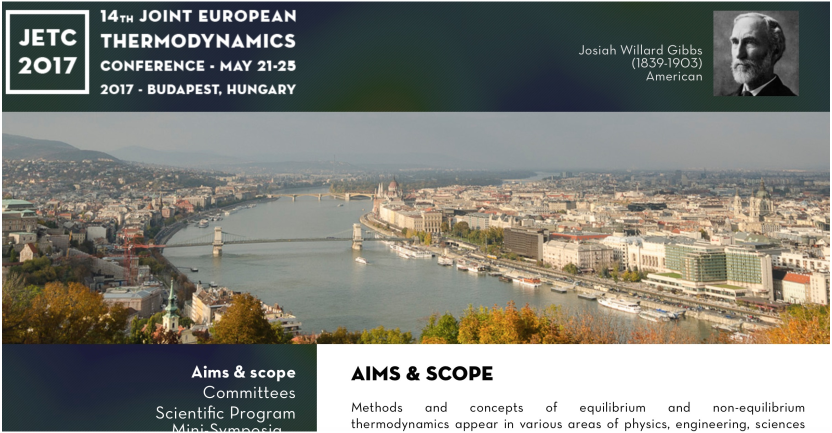
JETC 2017
14TH JOINT EUROPEAN THERMODYNAMICS CONFERENCE - MAY 21-25 2017 - BUDAPEST, HUNGARY
Josiah Willard Gibbs (1839-1903) American
Aims & scope
Committees
Scientific Program
AIMS & SCOPE
Methods and concepts of equilibrium and non-equilibrium thermodynamics appear in various areas of physics, engineering, sciences