Climate as a dynamical system

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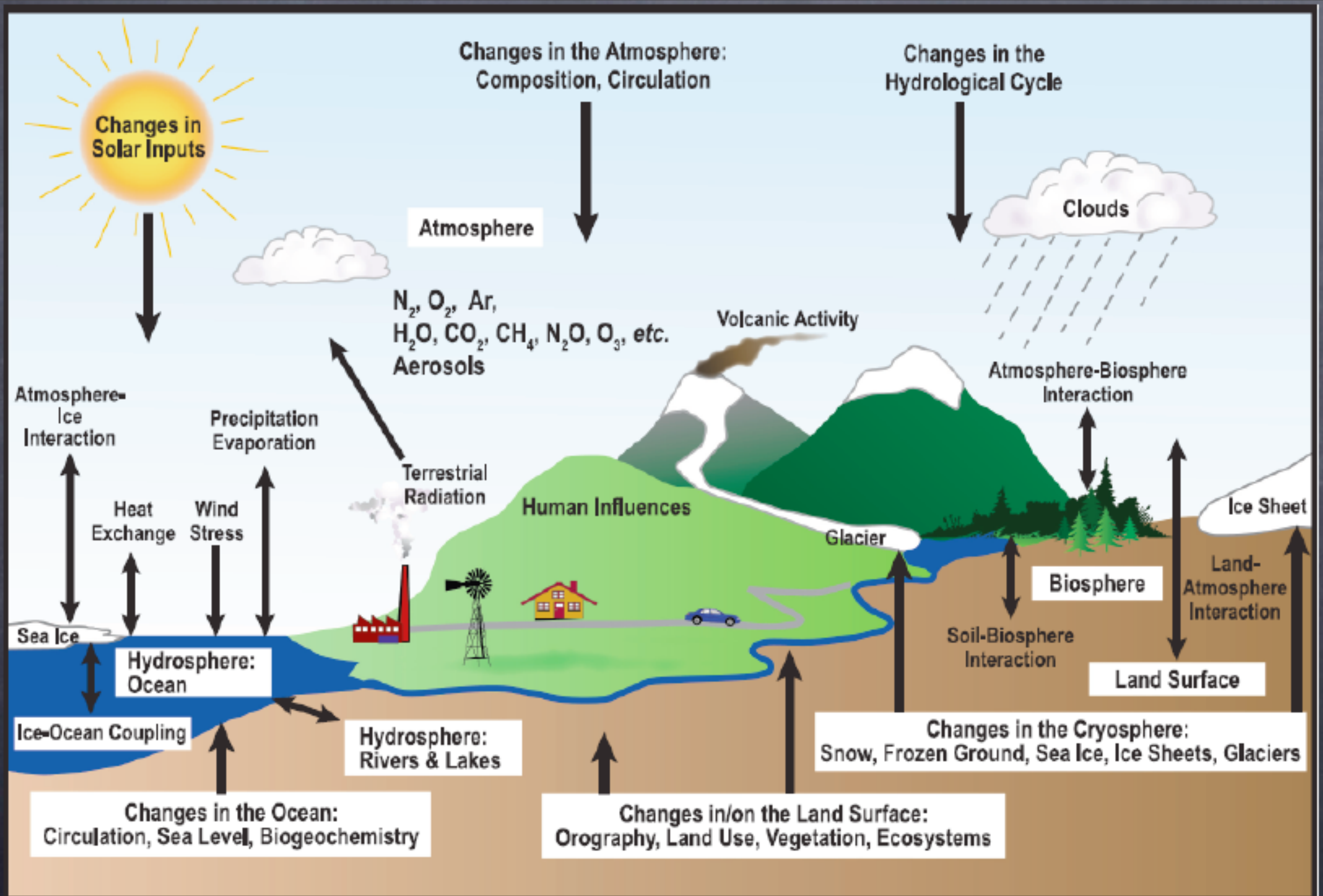
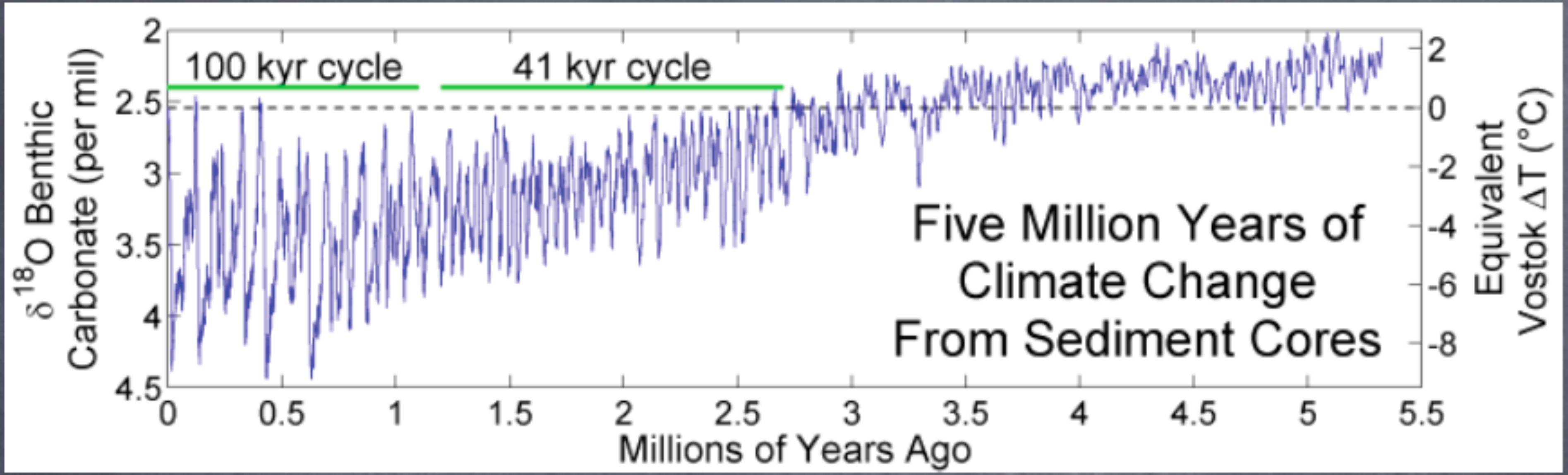
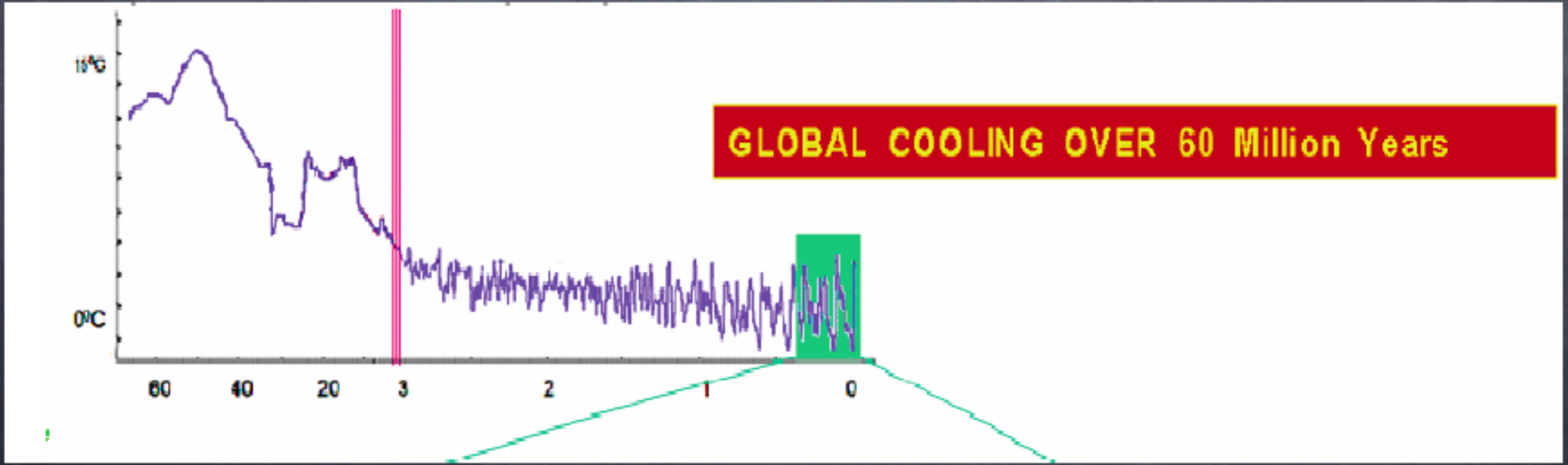
Climate: the statistical state of the Earth system

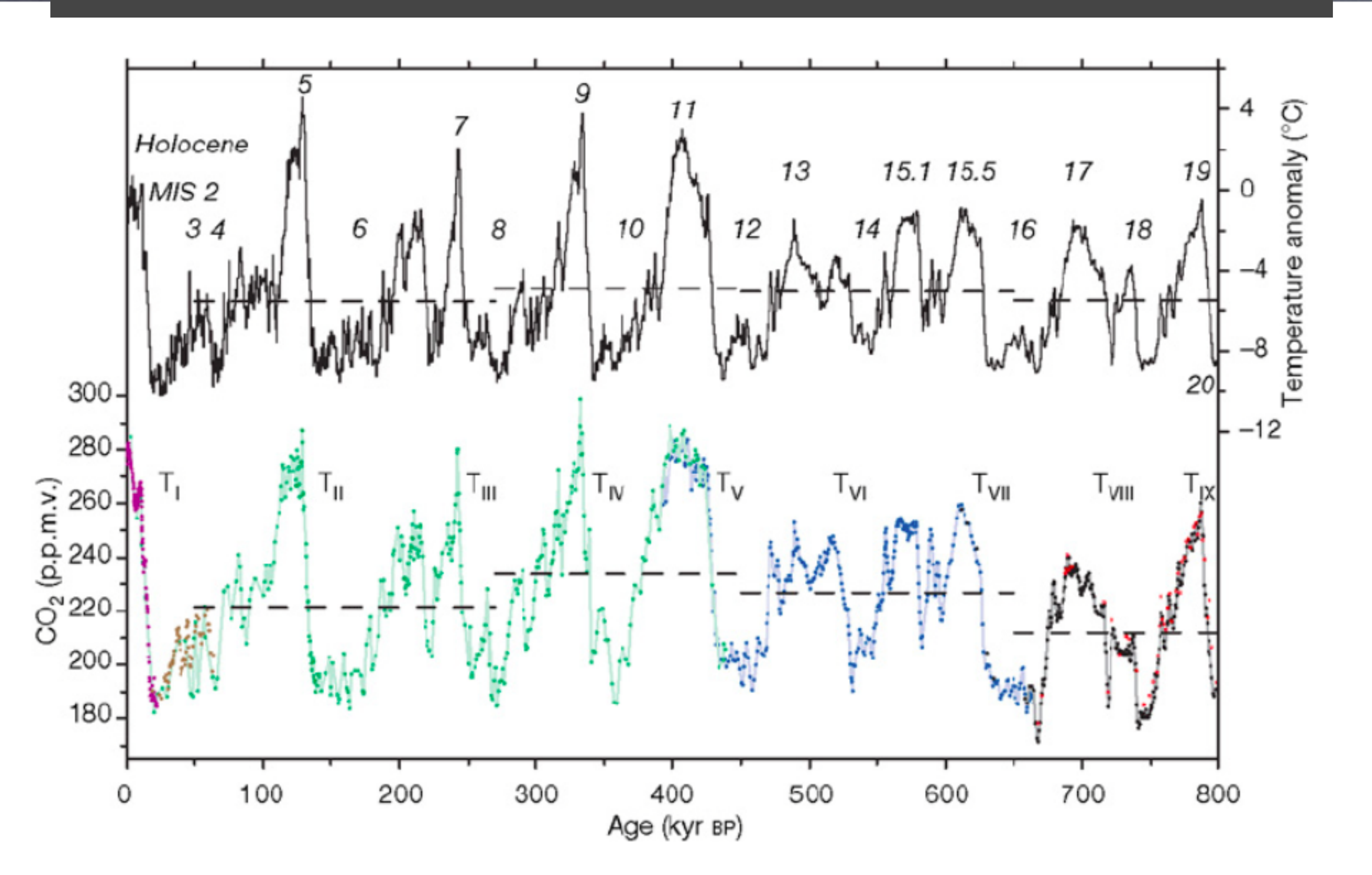


Climate varies on all time scales

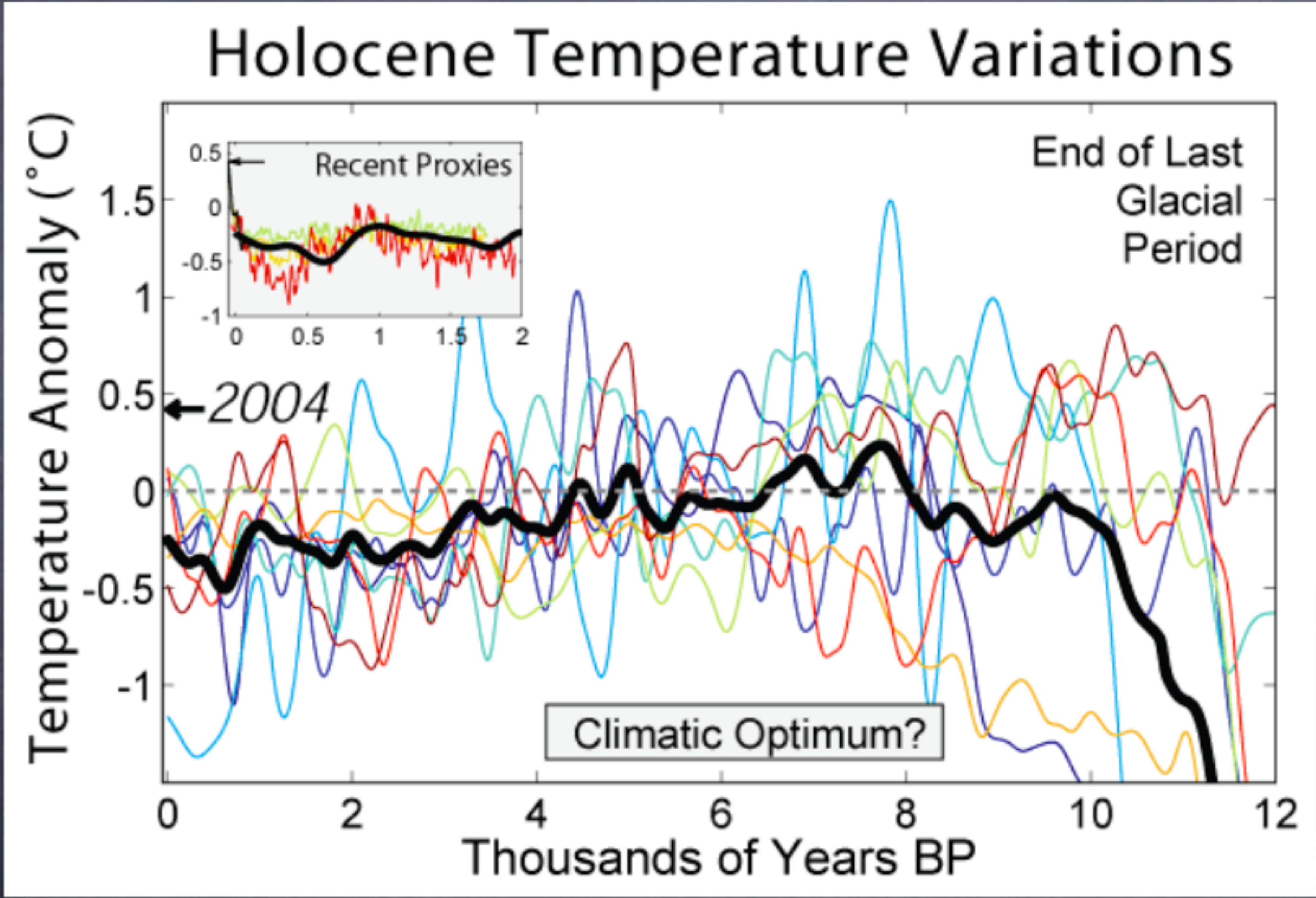




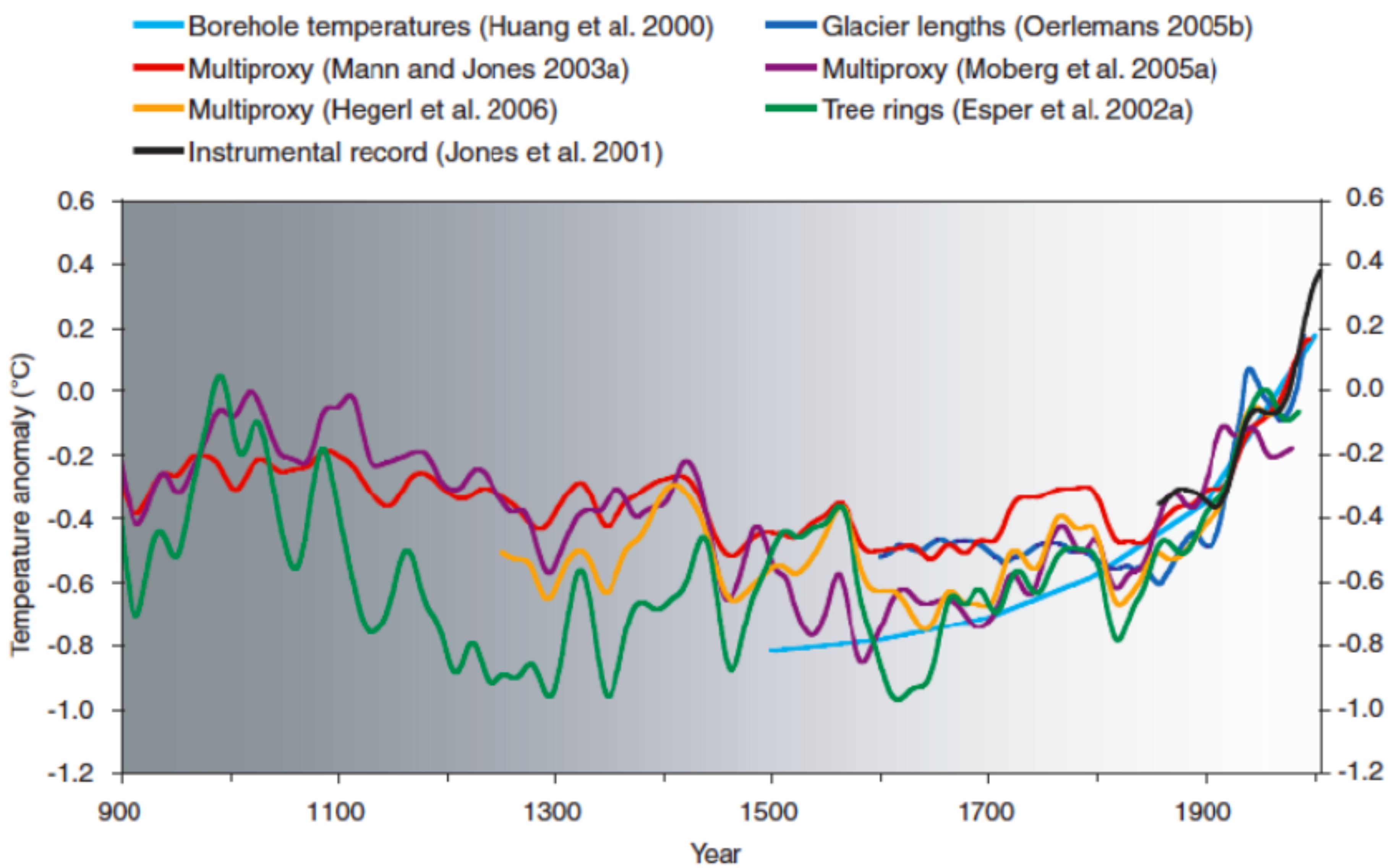
Climate varies on all time scales



Climate varies on all time scales



Besides natural variability, in the last 150 yr there are evidences of anthropogenic effects



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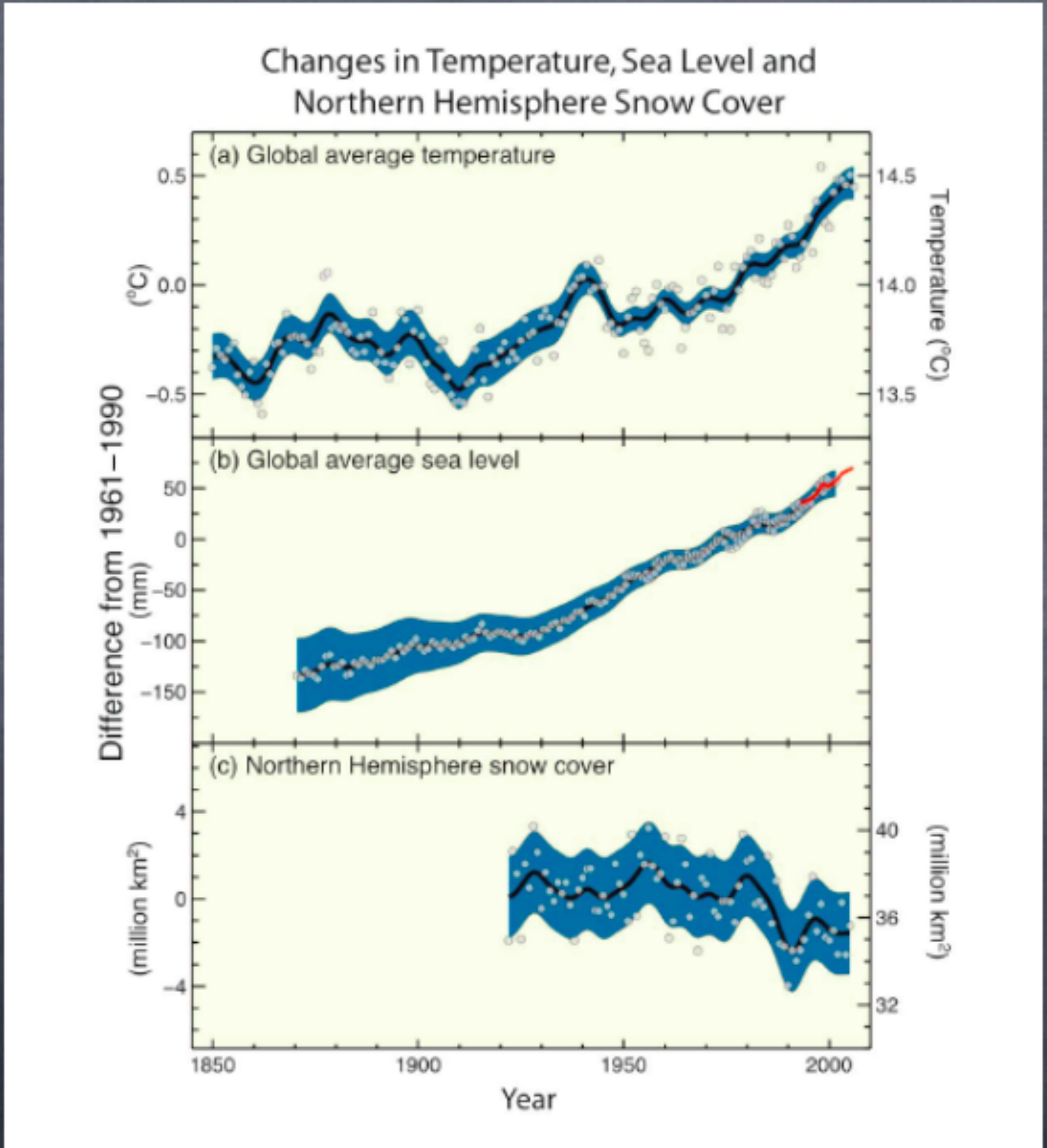
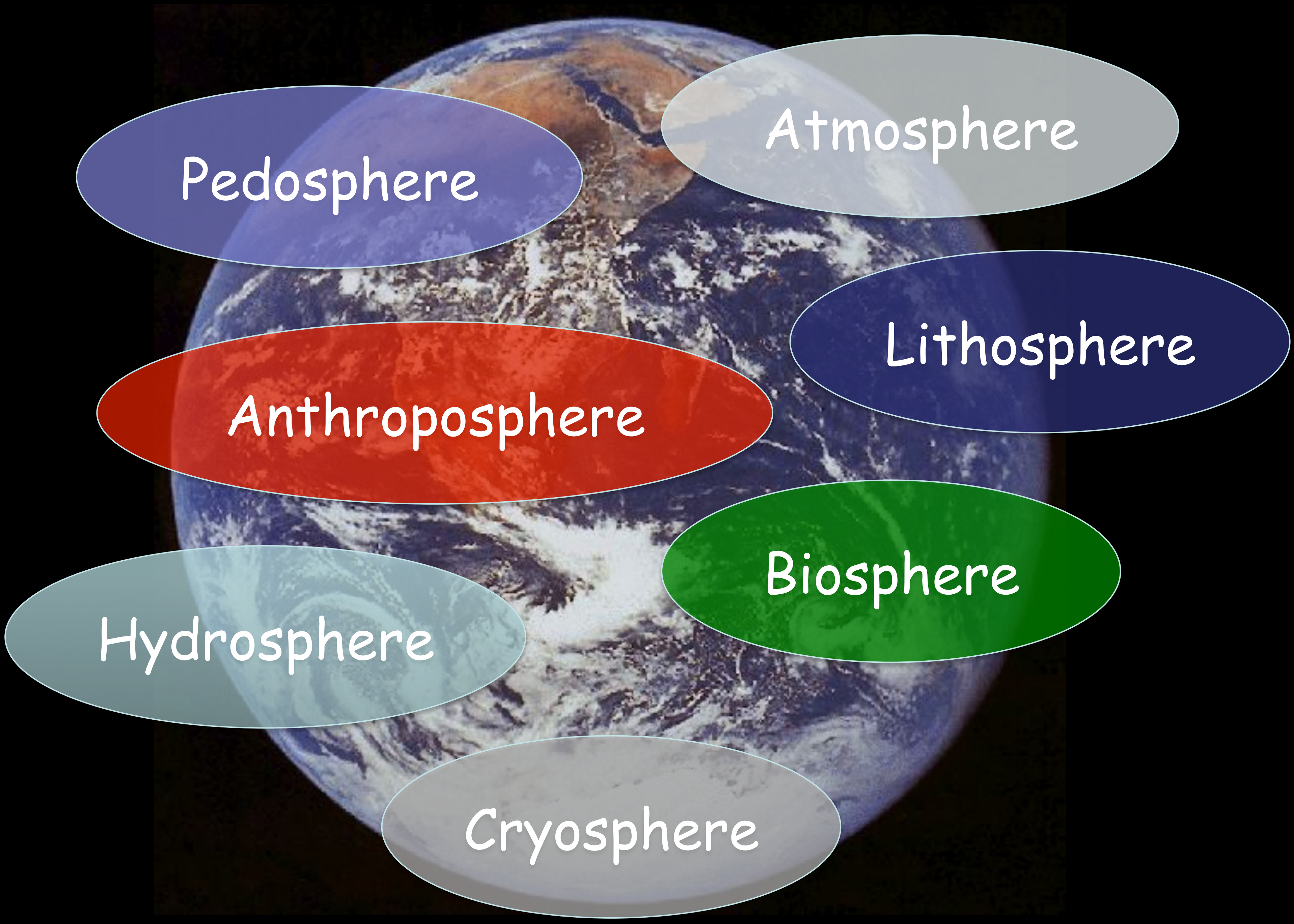
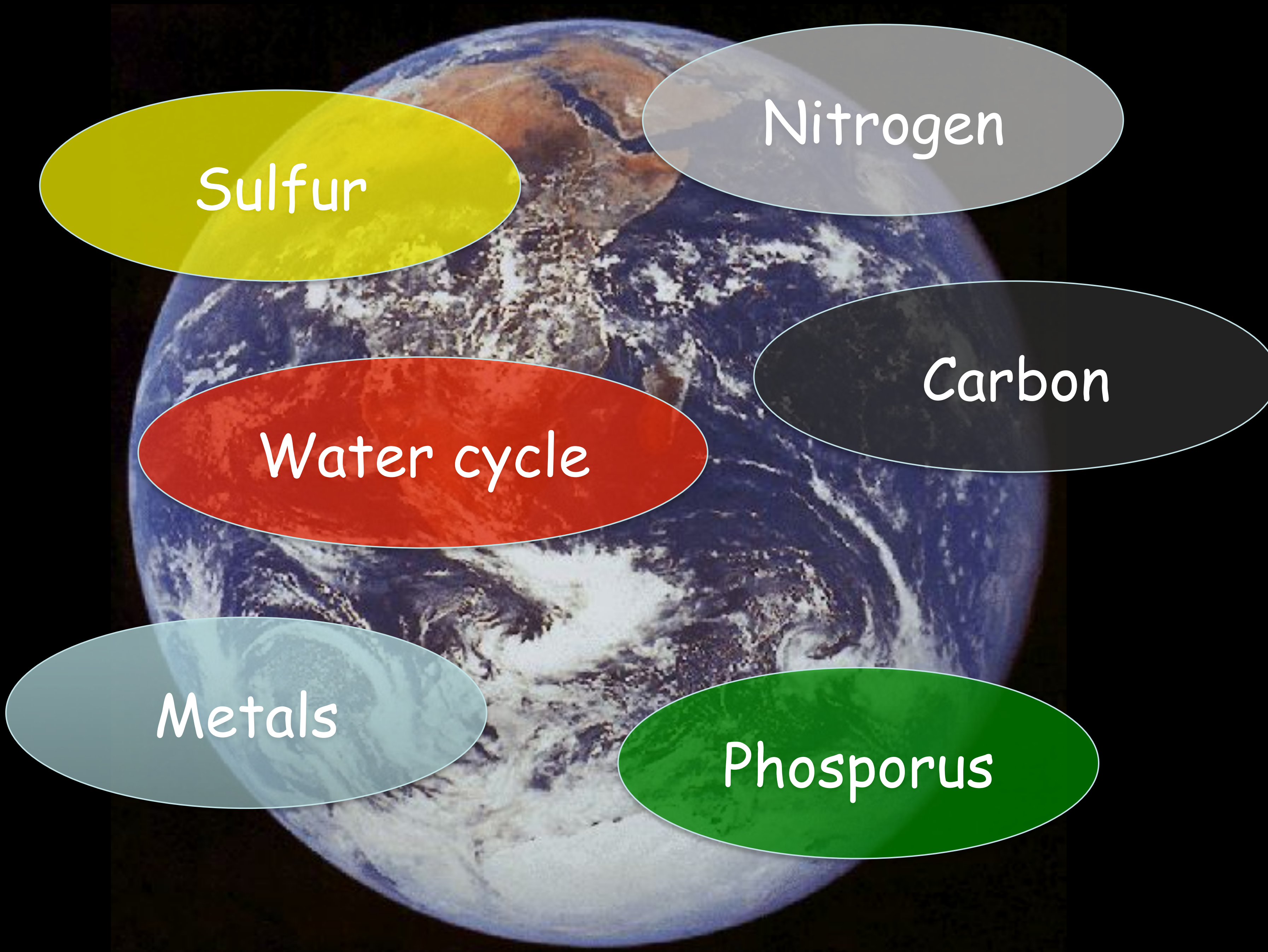


Climate is a complex system

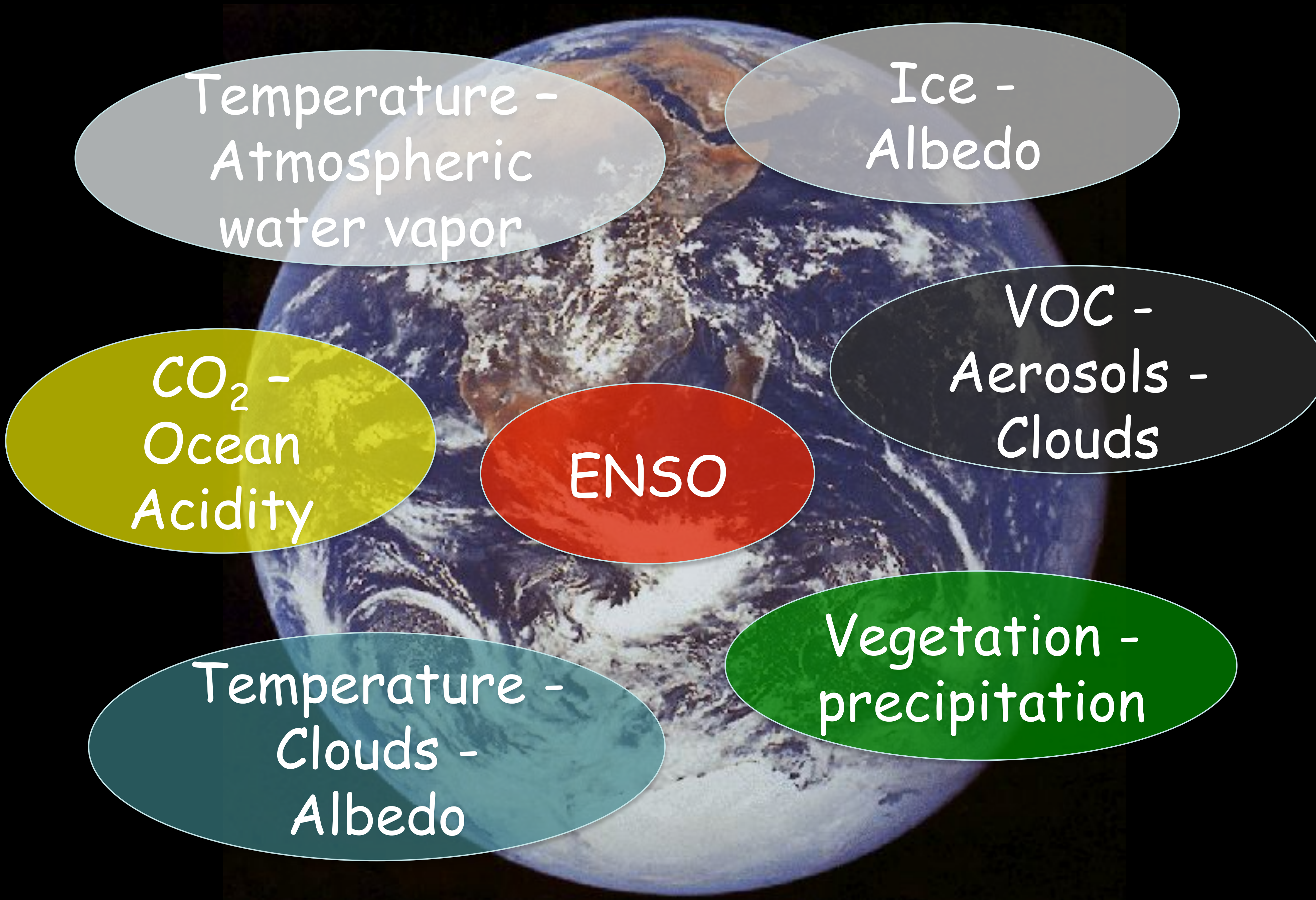
Reductionism I: the "spheres"



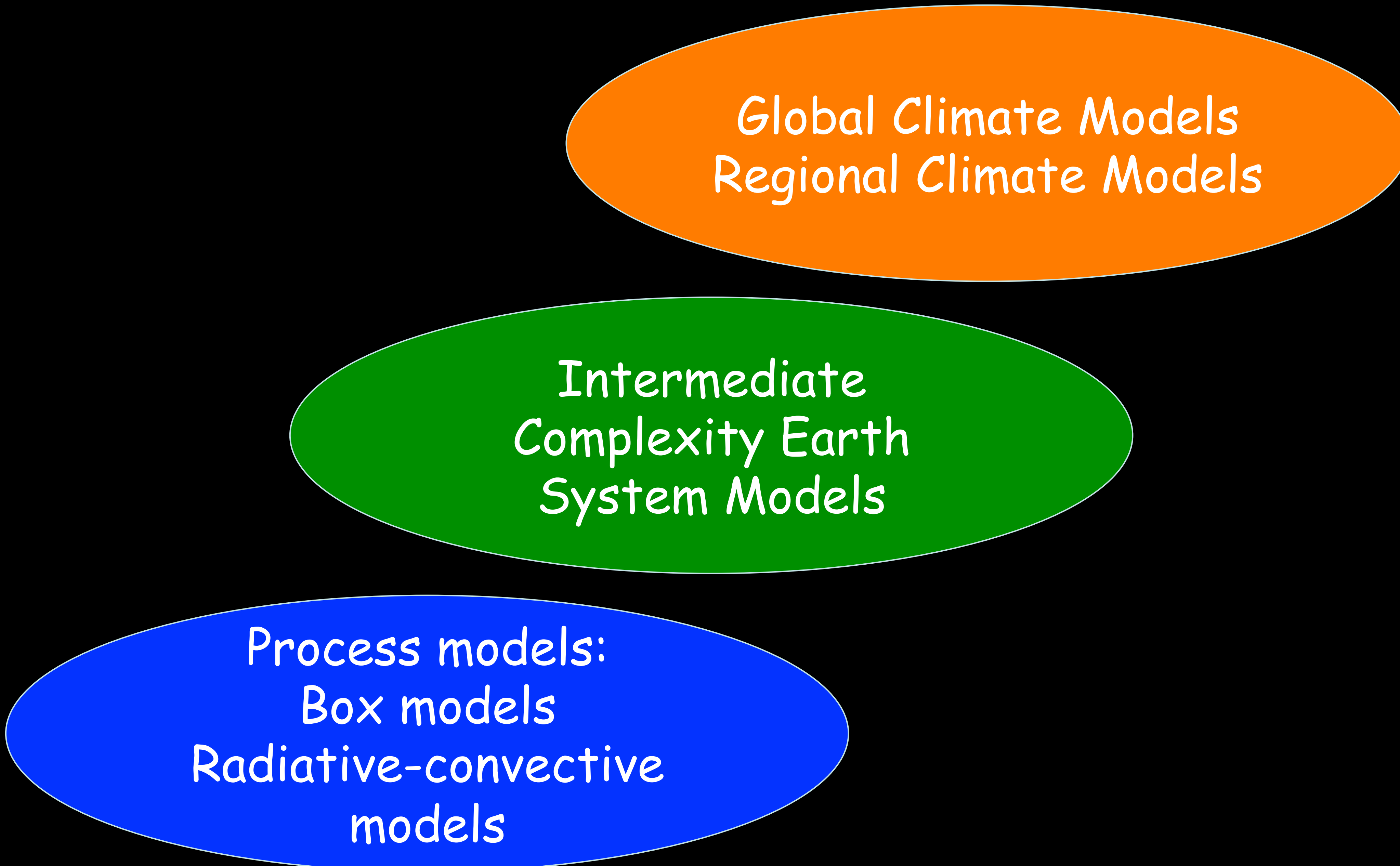
Reductionism II: biogeochemical cycles



Reductionism III: process decomposition



The climate model hierarchy



Conceptual models of climate:

Energy Balance Models

Midlatitude atmospheres
and the Lorenz 1983 model

Climate-biosphere interactions:

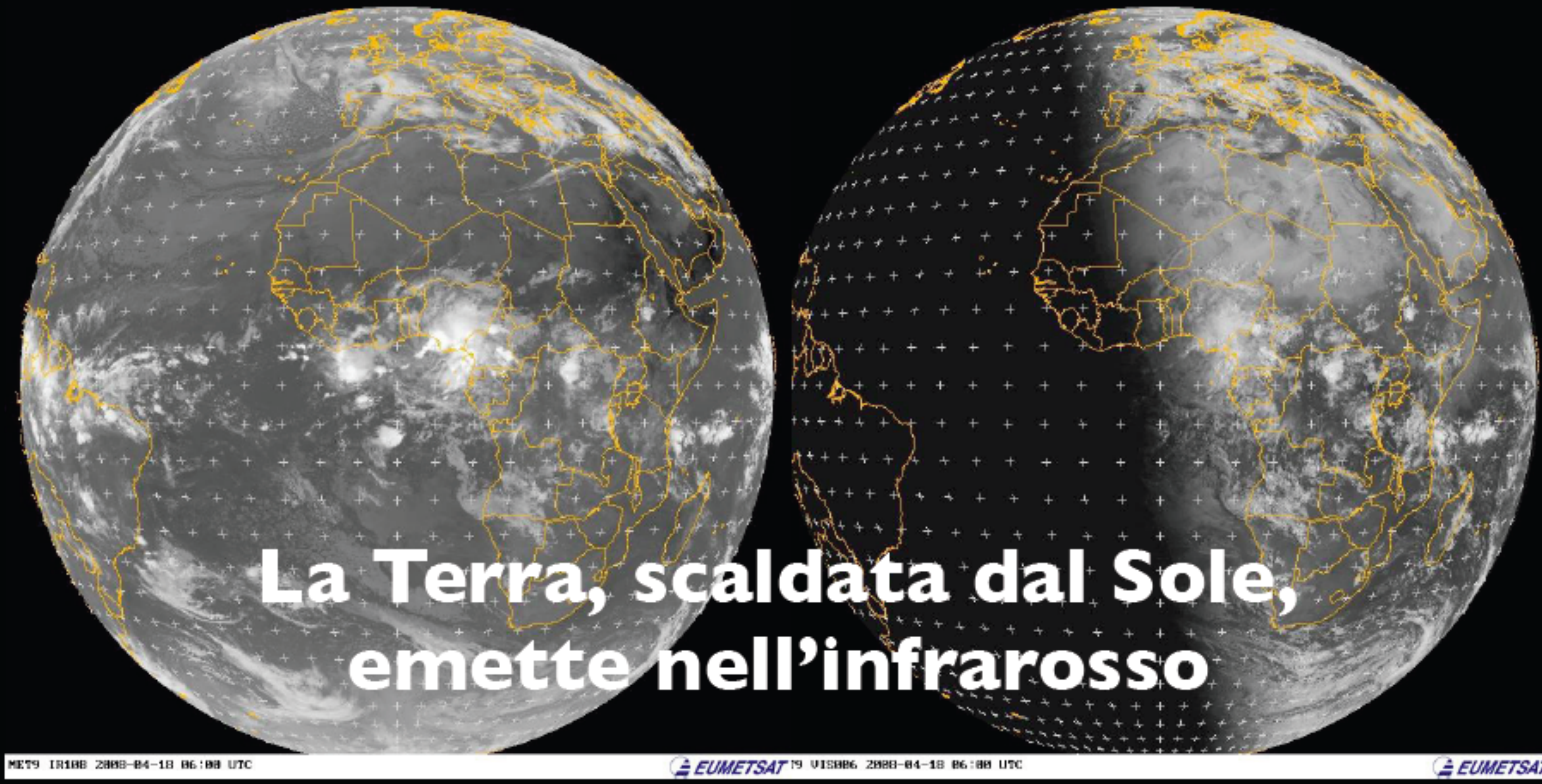
the Charney mechanism

Midlatitude summer droughts
Planet Dune

METEOSAT 2° Generation
18/4/2008 06:00 UTC

INFRAROSSO

VISIBILE



La Terra, scaldata dal Sole, emette nell'infrarosso

www.eumetsat.int

The main energy source of Earth's climate is solar radiation.

Solar "constant":
S = 1.368 kW per square meter
+- 3.5% owing to orbit ellipticity

Power that hits the top of Earth's atmosphere:

$$\pi S R^2$$

Surface over which it is distributed: $4\pi R^2$

Average power per unit surface: $\frac{1}{4}S$

Albedo:
Part of the incident energy is reflected

The fraction of reflected energy
is denoted by

$$\alpha$$

Fraction of absorbed power:

$$1-\alpha$$

On average, Earth's albedo is

$$\bar{\alpha}=0.3$$

Average absorbed power
(per unit surface):

$$P_{in} = \frac{1}{4} S\,(1 - \bar{\alpha}) \approx 240\, W\, m^{-2}$$

Emitted power (black body radiation)

$$P_{out} = \sigma T^4$$

$$\sigma = 5.7 \times 10^{-8}\; W\, m^{-2}\, K^{-4}$$

Stefan-Boltzmann constant

first principle of Thermodynamics

$$dE + dL = dQ$$

For the Earth as a point in the empty space

$$dE \approx C_V dT \ , \ \ dL = 0$$

C_V specific heat at constant volume

$$C_V \frac{dT}{dt} = P_{in} - P_{out}$$

$$C_V \frac{dT}{dt} = \frac{1}{4} S \left(1 - \bar{\alpha}\right) - \sigma \, T^4$$

If we look for a stationary state

$$\frac{1}{4} S\,(1-\bar{\alpha}) - \sigma\, T^4 = 0$$

$$T = \left[\frac{1}{4\sigma} S\,(1-\bar{\alpha})\right]^{1/4} \approx 255\,K$$

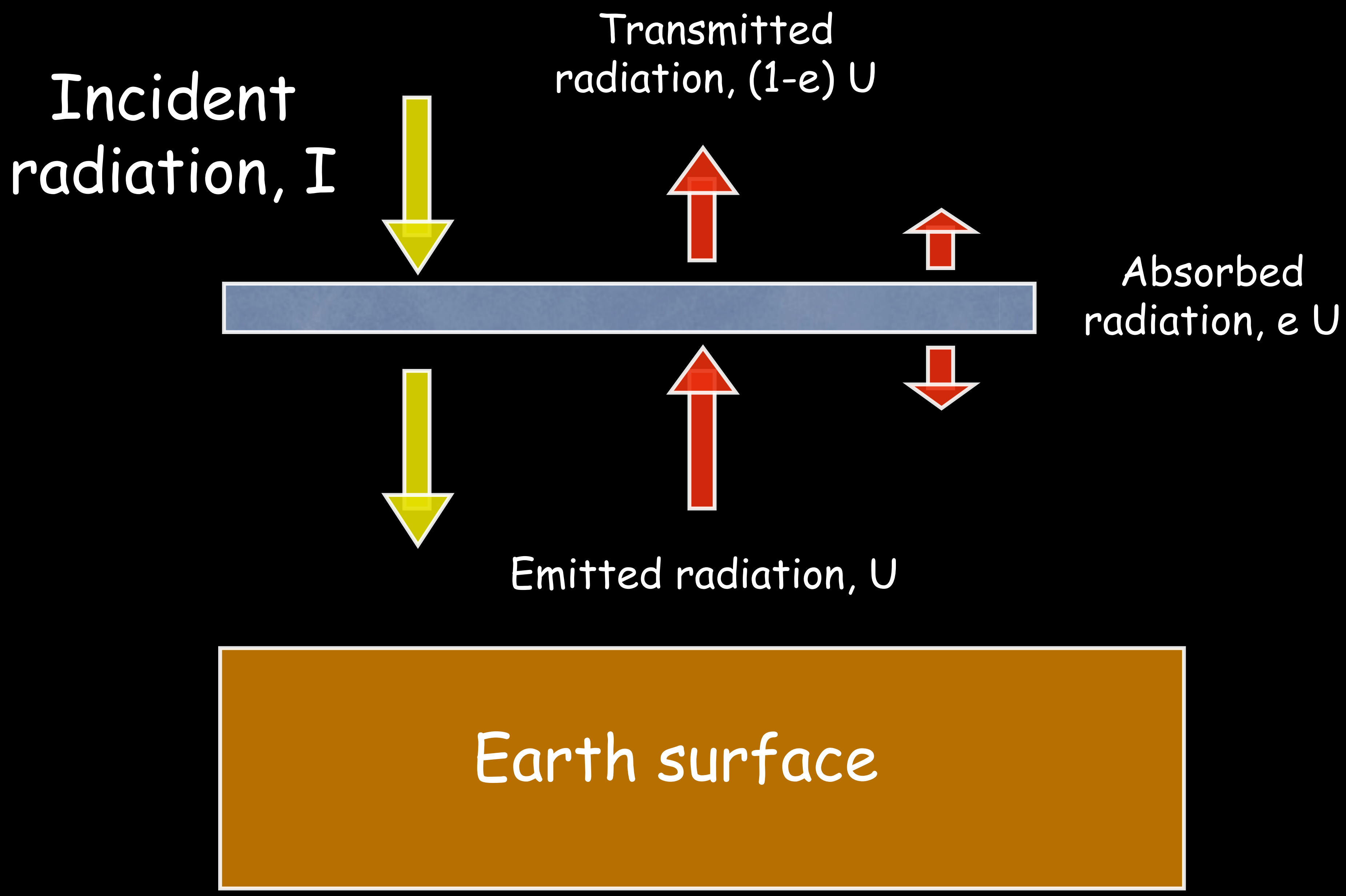
We can also look for a local equilibrium at latitude ϕ

$$T_{loc} = \left[\frac{1}{4\sigma}\,(1-\bar{\alpha})\,f(\phi)\right]^{1/4}$$

$$T_{eq} \approx 270\,K$$

$$T_{NP} \approx 170\,K \qquad T_{SP} \approx 150\,K$$

Role of the fluid envelope: greenhouse effect



Energy balance without greenhouse effect

$$\sigma T^4 = U = I$$

Greenhouse balance with greenhouse effect

$$I + \frac{e}{2}U = U$$

$$\sigma T^4 = U = \frac{I}{1 - e/2}$$

Spettro di radianza in onda lunga: REFIR-PAD, Teresina; Brasile, 30 giugno 2005

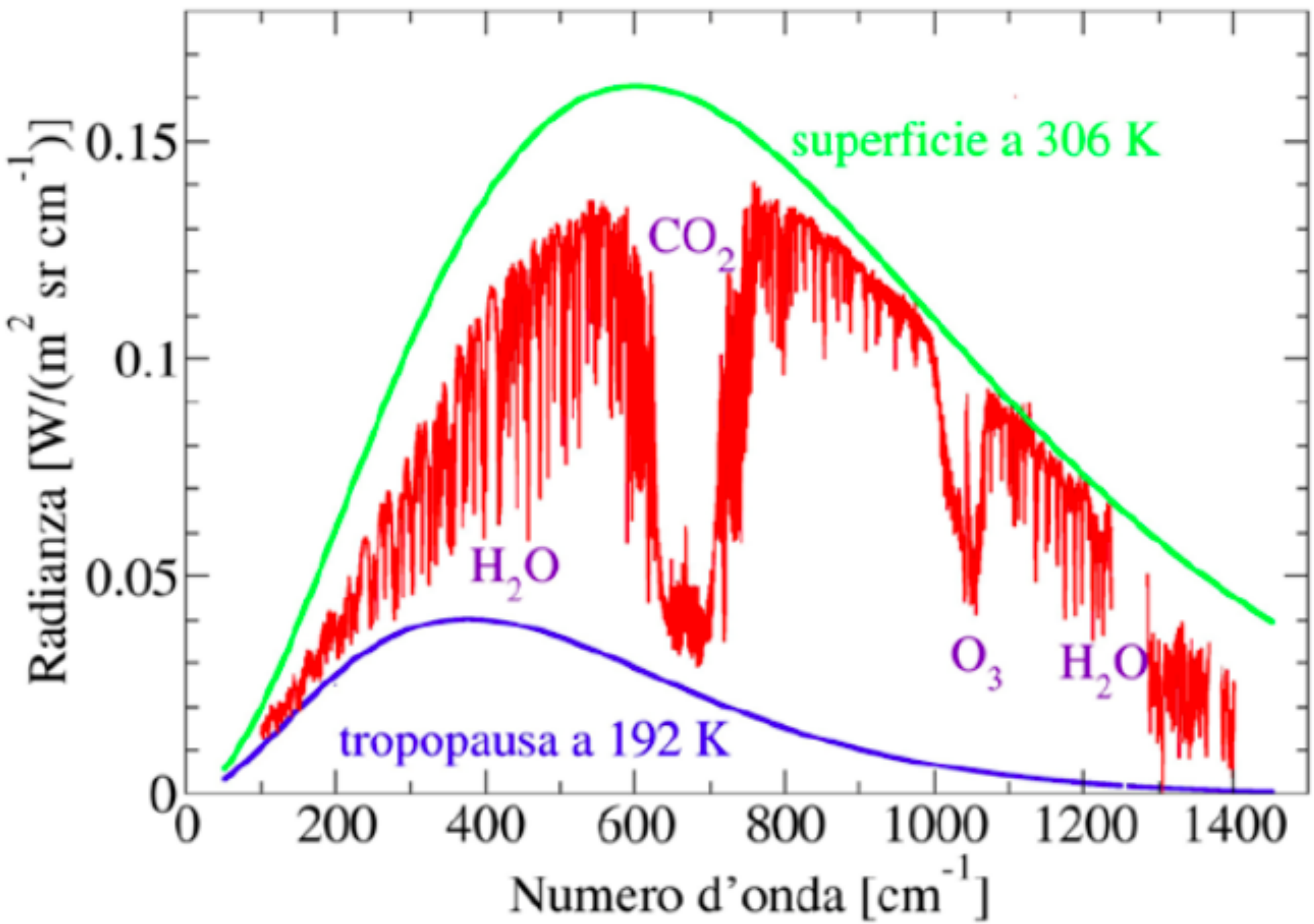


Figura 1a. Spettro di radianza in onda lunga (in Watt per m^2 per steradiante per cm^{-1}) con cui la Terra si raffredda disperdendo energia verso lo spazio, misurato da ricercatori dell'Istituto di Fisica Applicata del CNR con un esperimento effettuato in Brasile da pallone stratosferico a 34 km di quota. Curva verde: emissione di corpo nero della superficie terrestre a 306 K (gradi Kelvin) (circa +32.85 °C); curva blu: emissione di corpo nero della tropopausa a 192 K (circa -81.15 °C); curva rossa: radianza misurata nell'esperimento. Si tratta della prima misura spettralmente risolta di tutta la radianza includendo anche le componenti a grande lunghezza d'onda. Si nota, in particolare, il contributo dei principali gas che causano l'effetto serra. (Riadattata da Palchetti et al., *Atmos. Chem. Phys* 2006.)

Effetto serra: REFIR-PAD, Teresina; Brasile, 30 giugno 2005

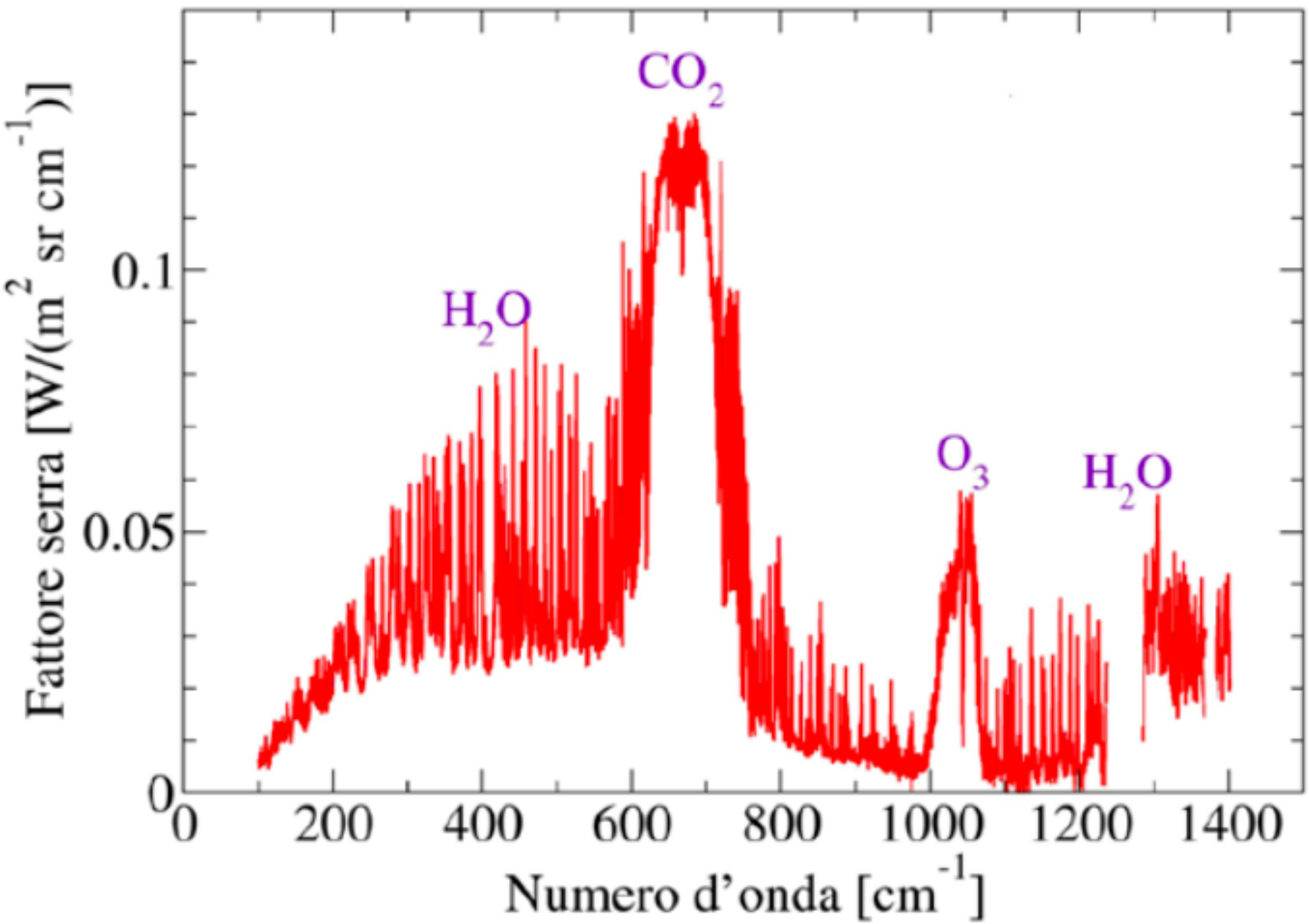


Figura 1b. Quantificazione dell'effetto serra, ottenuta dalla differenza fra l'emissione della superficie terrestre (curva verde in figura 1a) e l'emissione misurata in stratosfera (curva rossa in figura 1a). Le misure sono state effettuate da ricercatori dell'Istituto di Fisica Applicata del CNR con un esperimento effettuato da pallone stratosferico. (Riadattata da Palchetti et al., *Atmos. Chem. Phys* 2006.)

What determines the albedo of the Earth?

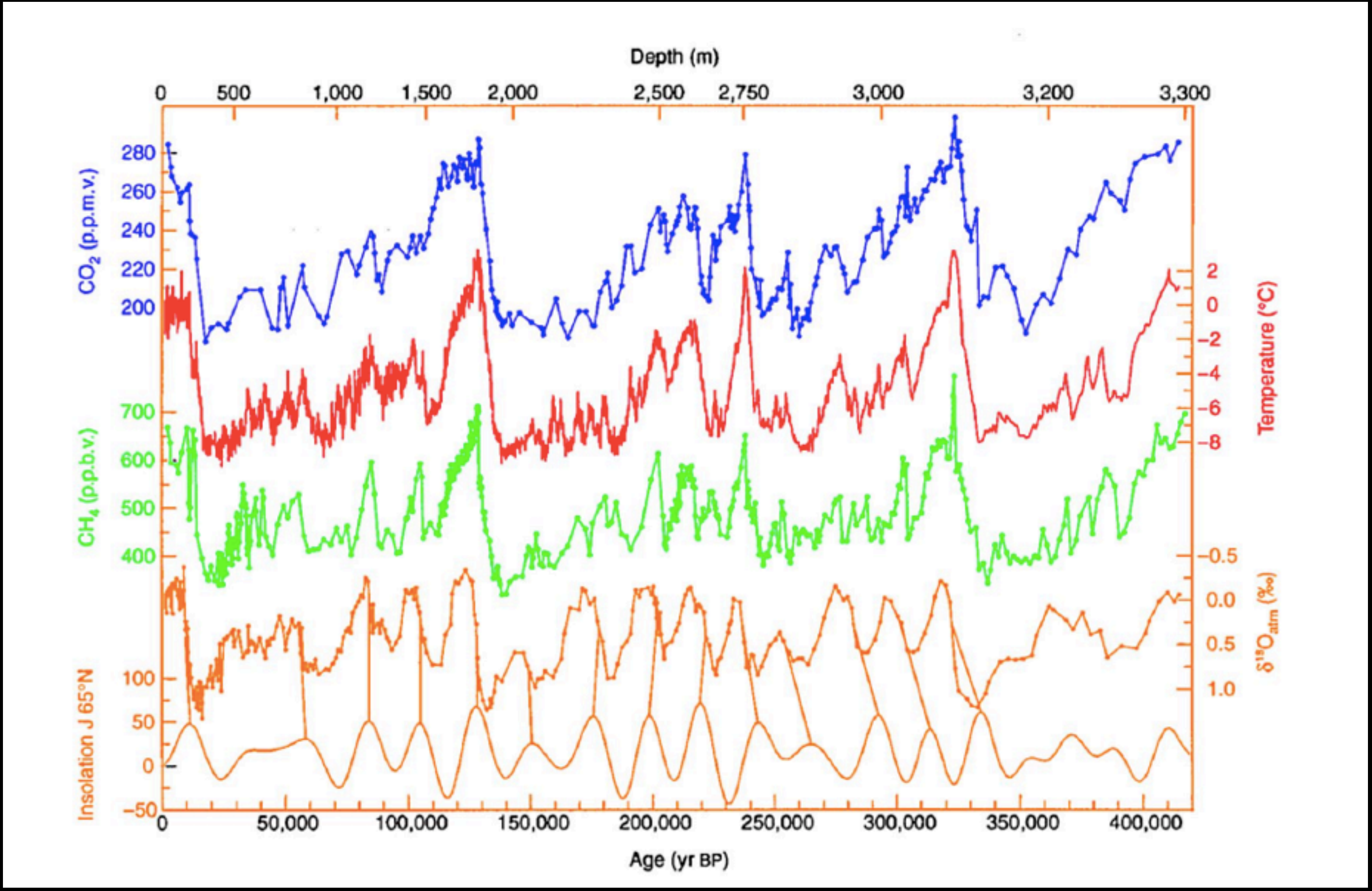
High albedo: snow and ice surfaces

High albedo: clouds

Medium albedo: desert sands, barren land

Low albedo: forests, ocean

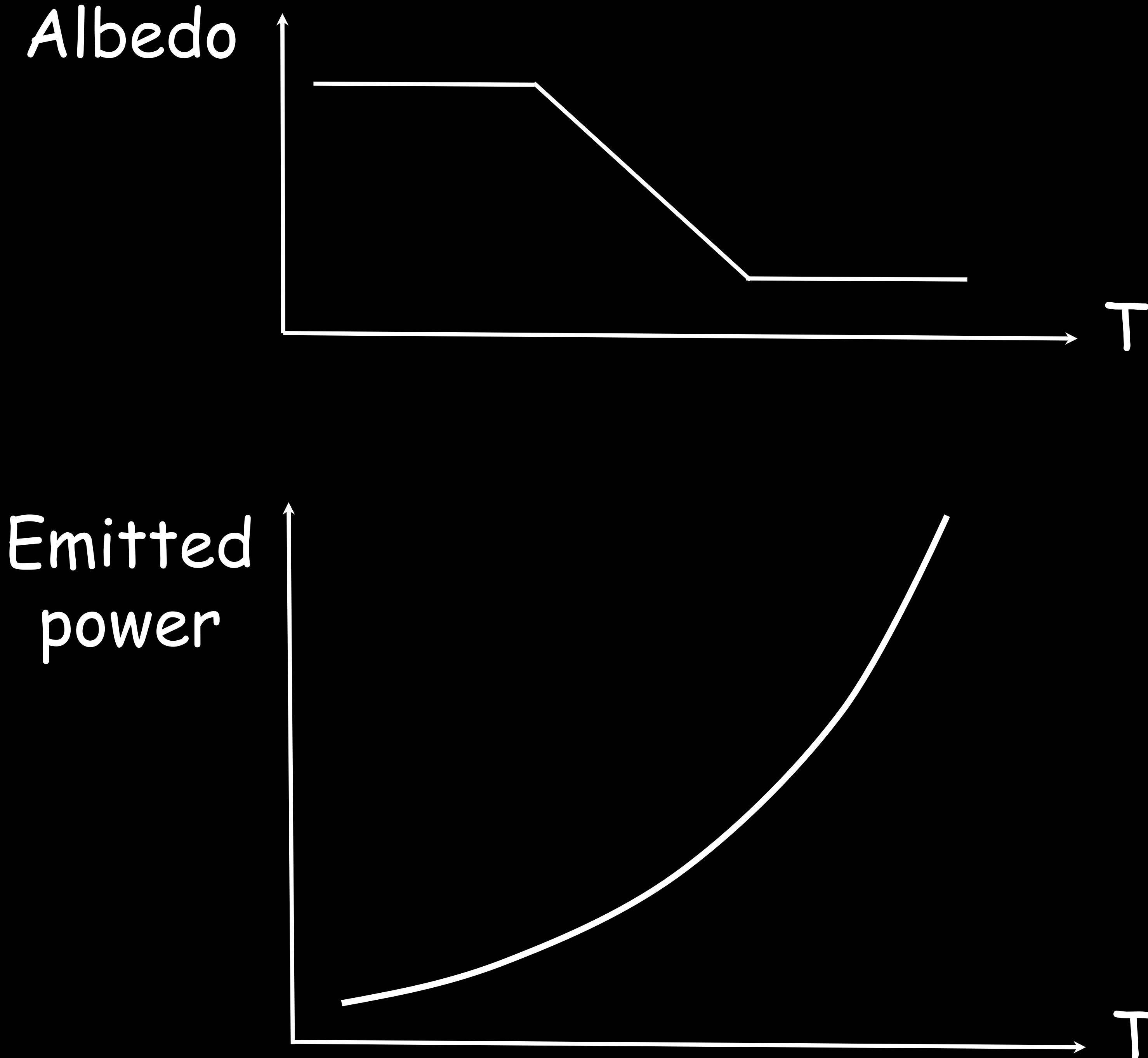
Quaternary glaciations and multiple equilibria of the climate system



Can we find a simple razionalization
of the two states (glacial and interglacial)
of the Earth's climate ?

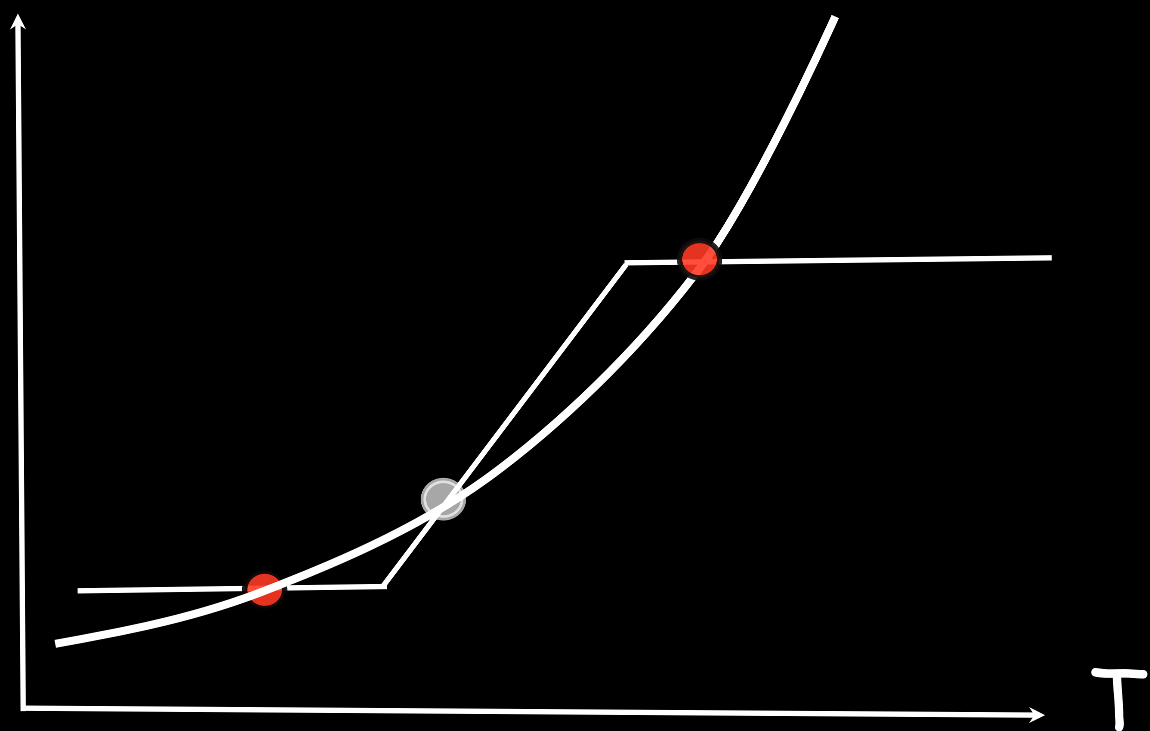
Ice-albedo feedback:

more ice, higher albedo
higher albedo, less absorbed heat
less absorbed heat, lower temperature
lower temperature, more ice


Albedo
T
Emitted
power
T

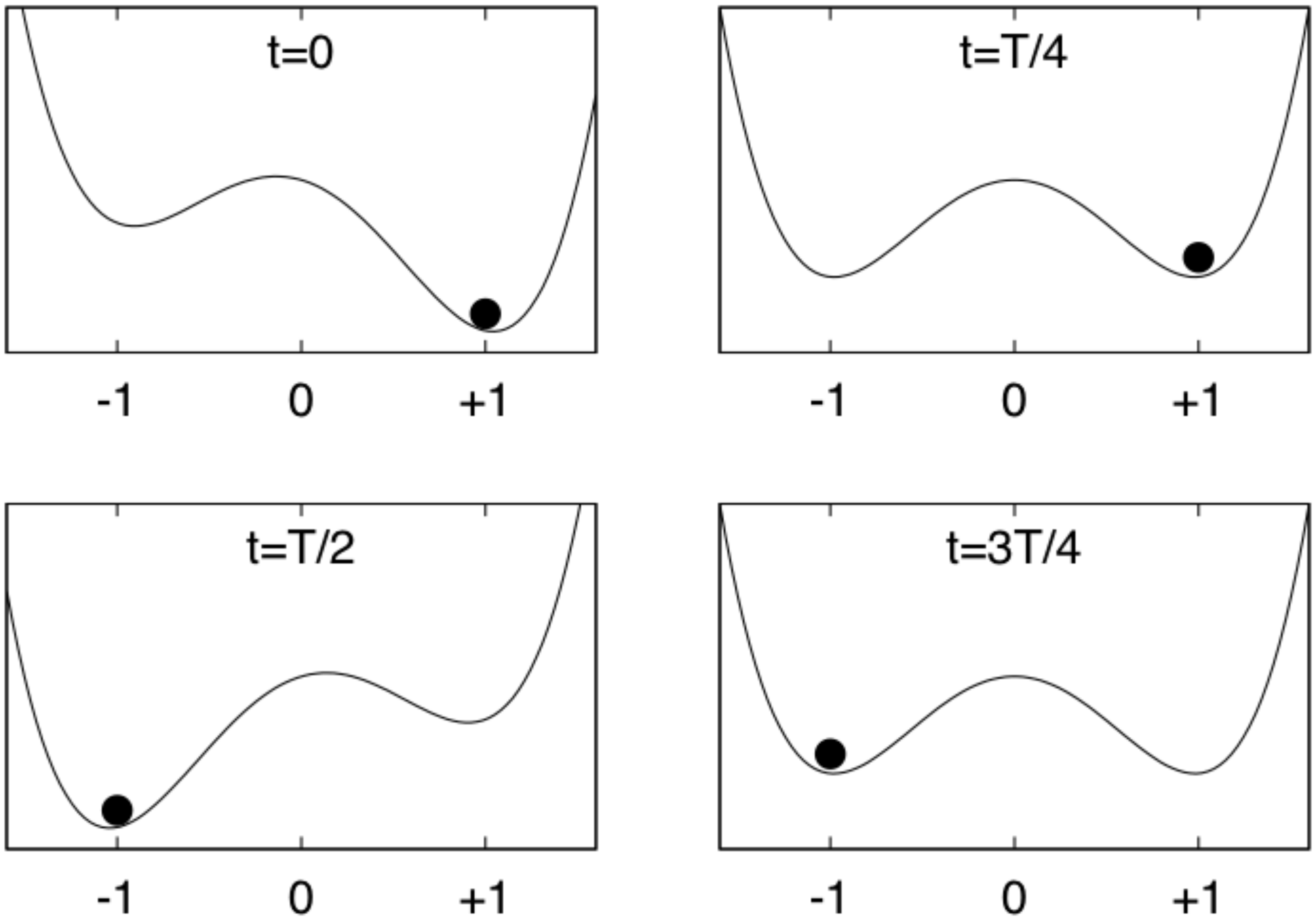
first principle of Thermodynamics

$$C_V \frac{dT}{dt} = \frac{1}{4} S \left(1 - \bar{\alpha}\right) - \sigma T^4$$

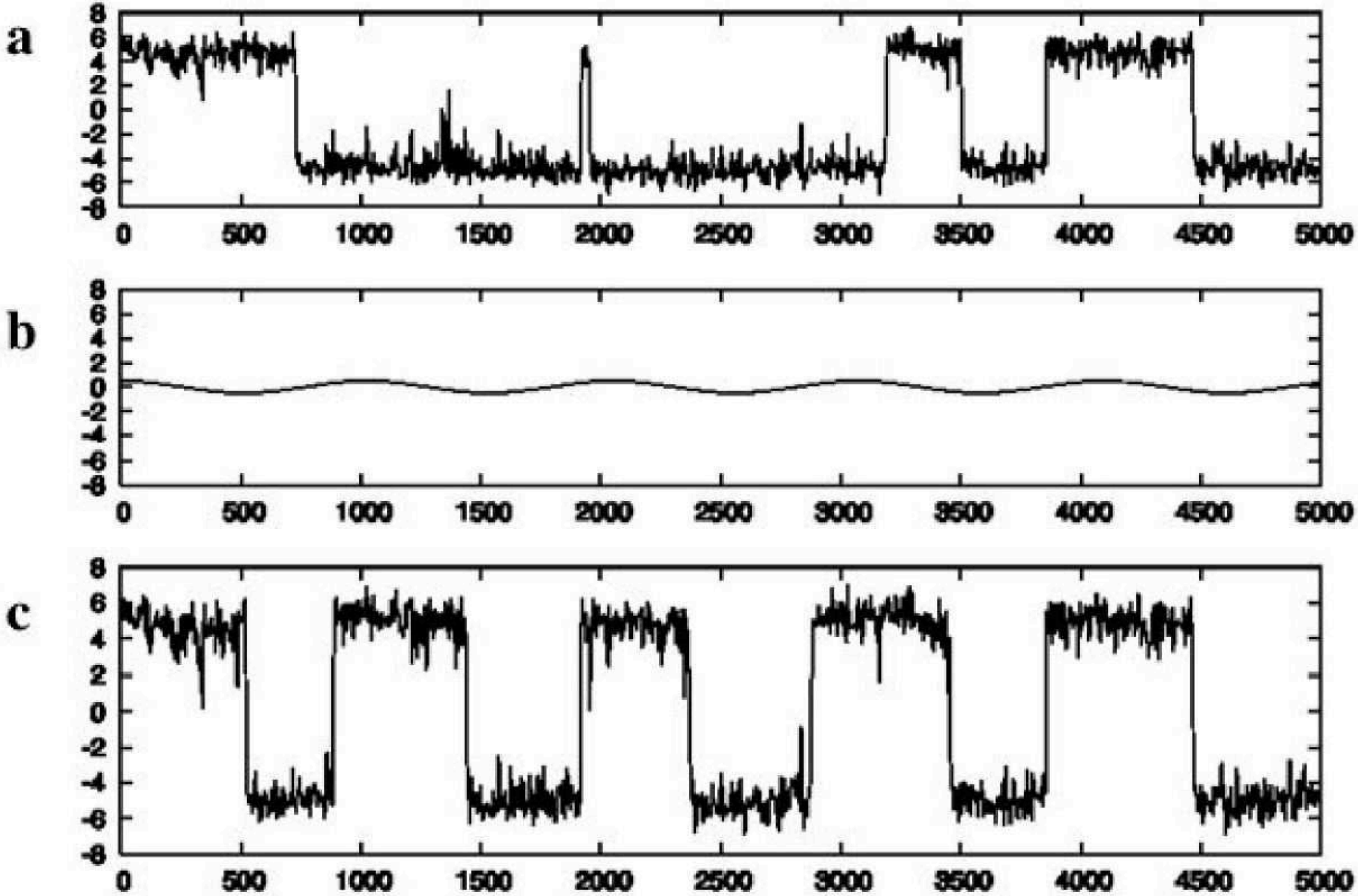


equivalent to overdamped motion in a double potential well

The ancient story of stochastic resonance



$$C_V \frac{dT}{dt} = \frac{S\left(1+\varepsilon \sin \Omega t\right)}{4}\left[1-\alpha(T)\right]+\sqrt{\sigma}\, W(t)$$



$$\tau \cong \tau_0 e^{\frac{2\Delta V}{\sigma}}$$

Another use of box models: feedbacks in the vegetation-climate system

feedbacks in the vegetation-climate system

A classic example: the Charney mechanism (1975):

Reduction in vegetation

↓

Higher albedo

↓

Local cooling of the soil

↓

Higher atmosph. pressure, reduced convection

↓

Lower precipitation

A classic example: the Charney mechanism (1975):

$$\frac{dV}{dt} = gV\left(1 - V\right) - mV$$

$$g = g(P) \quad , \quad P \propto T$$

Vegetation dynamics:

a logistic equation
for the fraction of soil
covered by vegetation, V

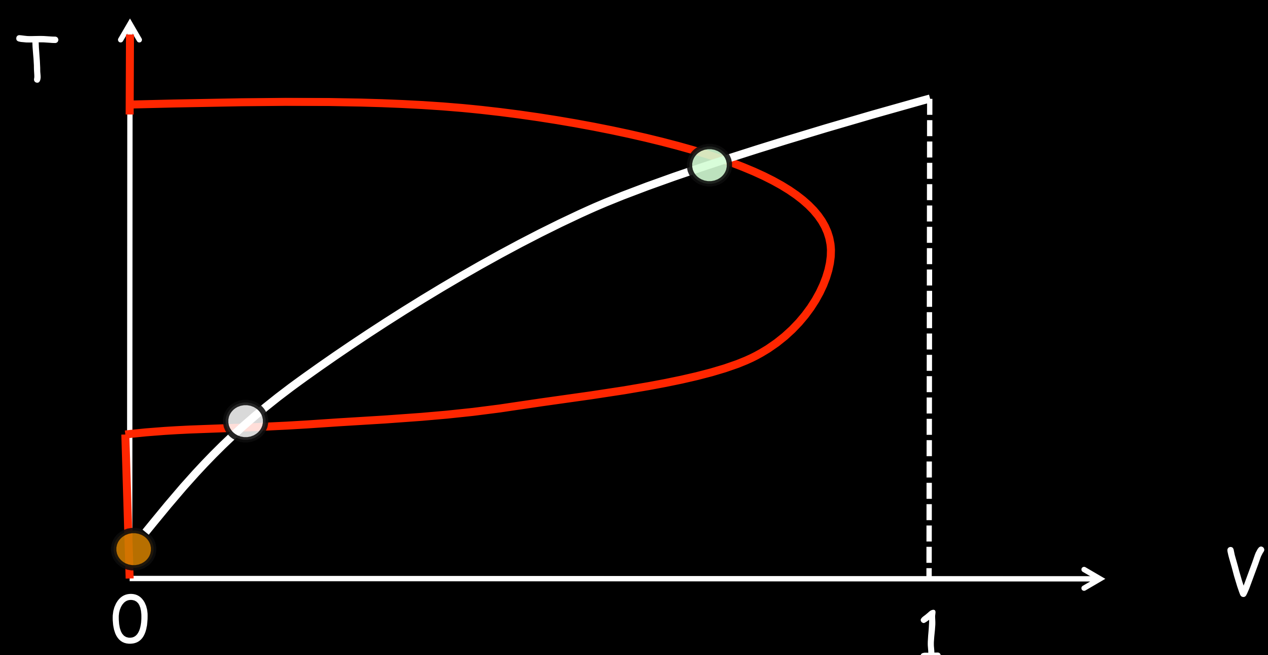
$$C_V \frac{dT}{dt} = \frac{S}{4}\left[1 - \alpha_V V - \alpha_B \left(1 - V\right)\right] - \sigma T^4$$

First principle
of Thermodynamics

A classic example: the Charney mechanism (1975):

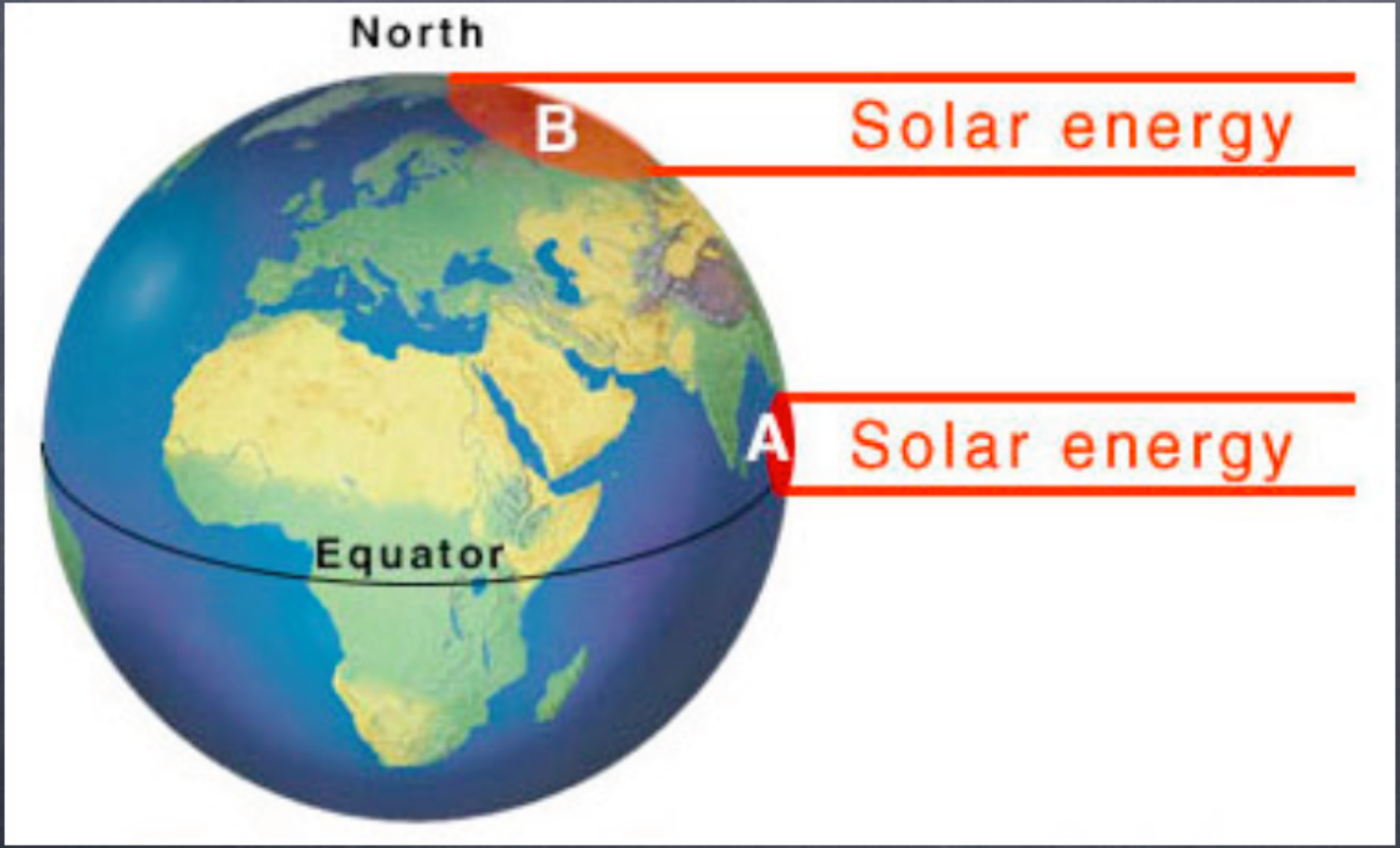
$$0 = g(T)V(1-V) - mV \quad \Rightarrow \quad V = 1 - \frac{m}{g(T)}$$

$$0 = \frac{S}{4}\left[1 - \alpha_V V - \alpha_B(1-V)\right] - \sigma T^4 \quad \Rightarrow \quad T = \sqrt[4]{\frac{S}{4\sigma}\left[1 - \alpha_V V - \alpha_B(1-V)\right]}$$



Beyond a point-wise Earth: latitudinal dependence

Solar energy is not evenly distributed on Earth

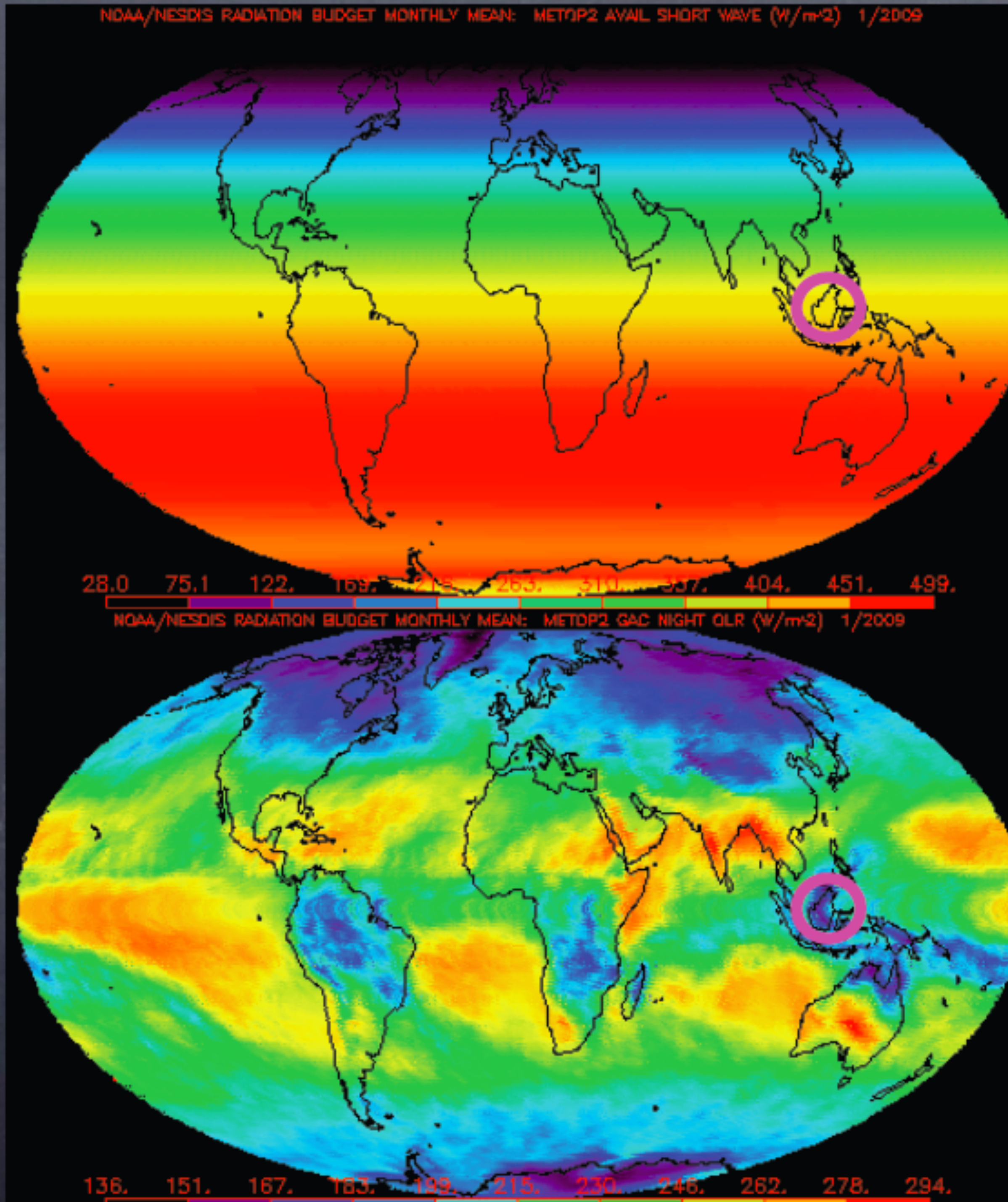


Local equilibrium at latitude ϕ

$$T_{loc} = \left[\frac{1}{4\sigma} (1 - \bar{\alpha}) \, f(\phi) \right]^{1/4}$$

$$T_{eq} \approx 270\,K$$

$$T_{NP} \approx 170\,K \qquad T_{SP} \approx 150\,K$$



IN
(SOLARE)

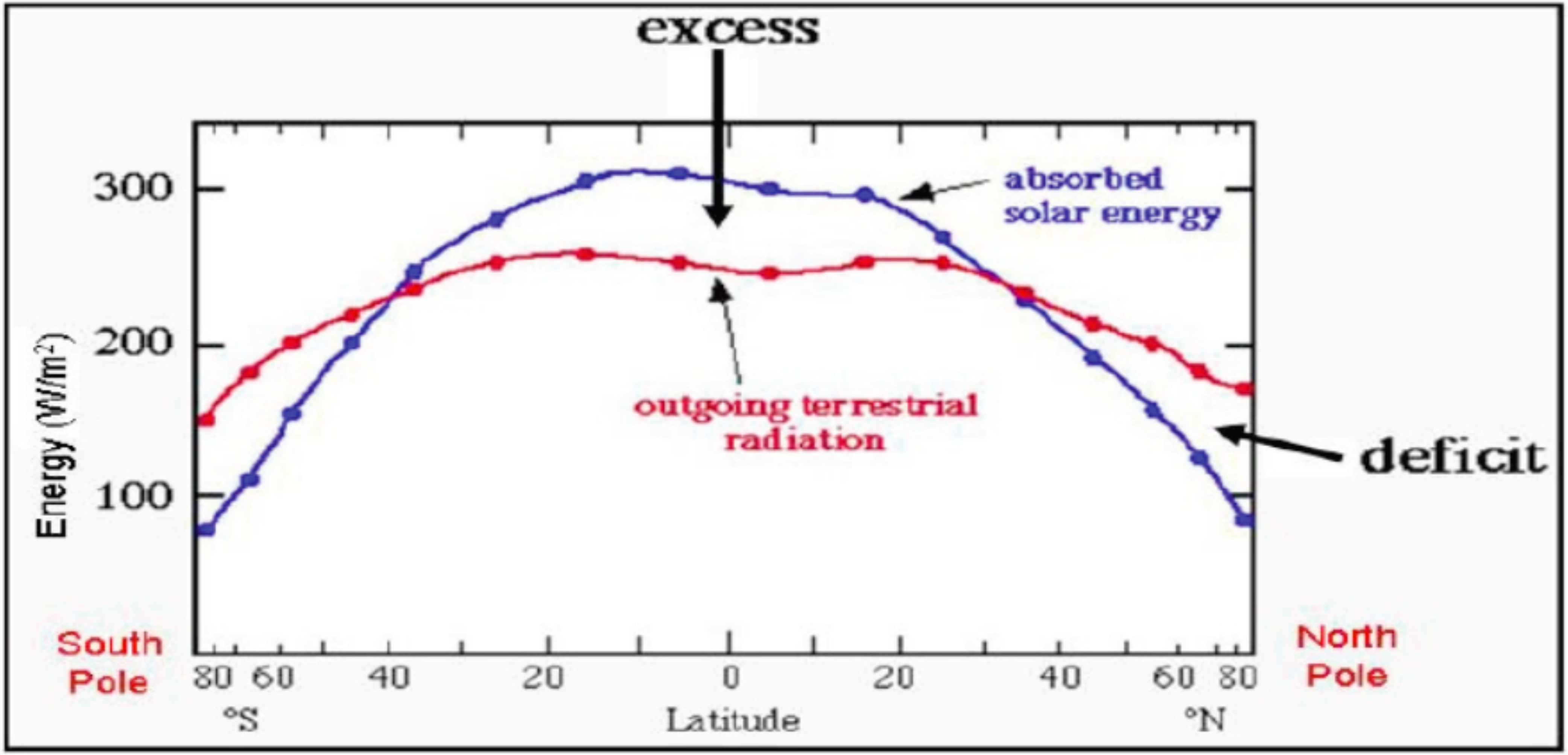
~410 W/m^2

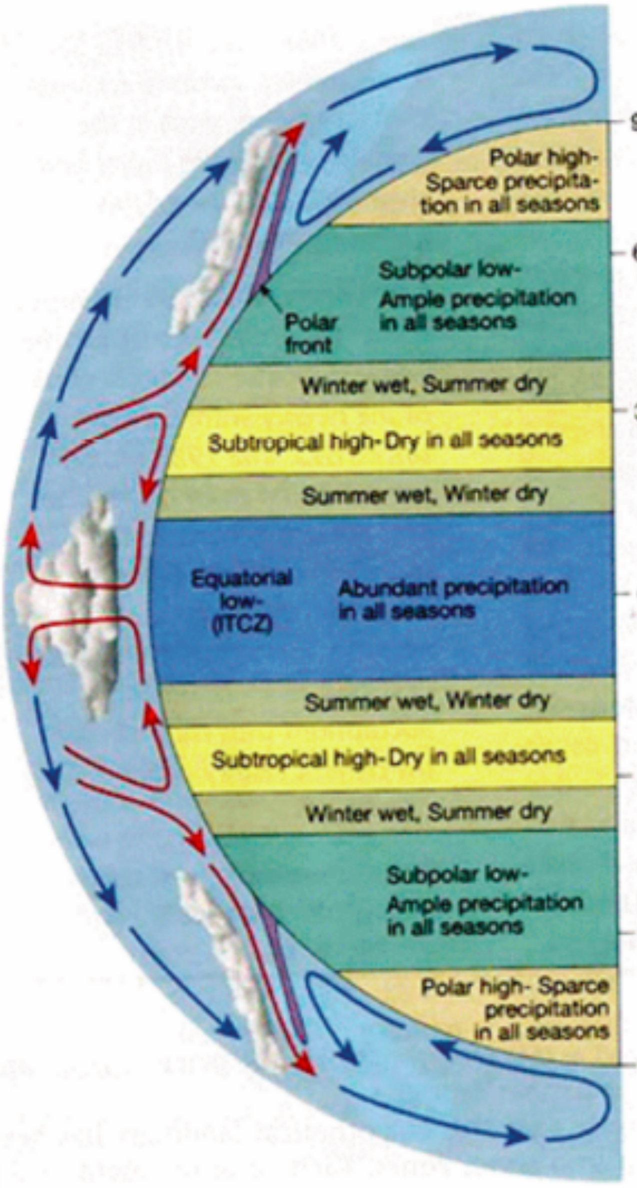
Energy does not go out from where it came in

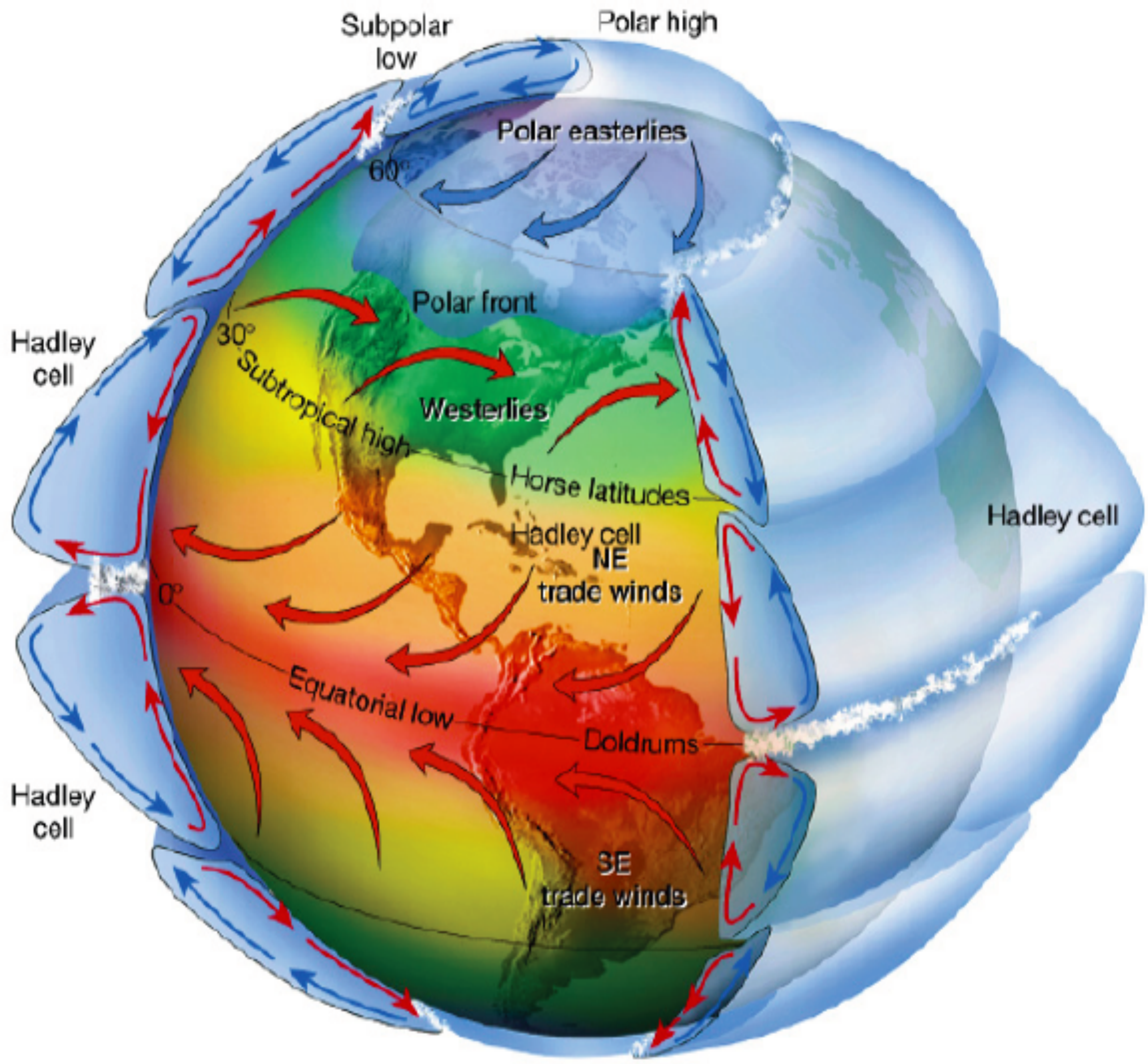
~170 W/m^2

OUT
(INFRAROSSO)

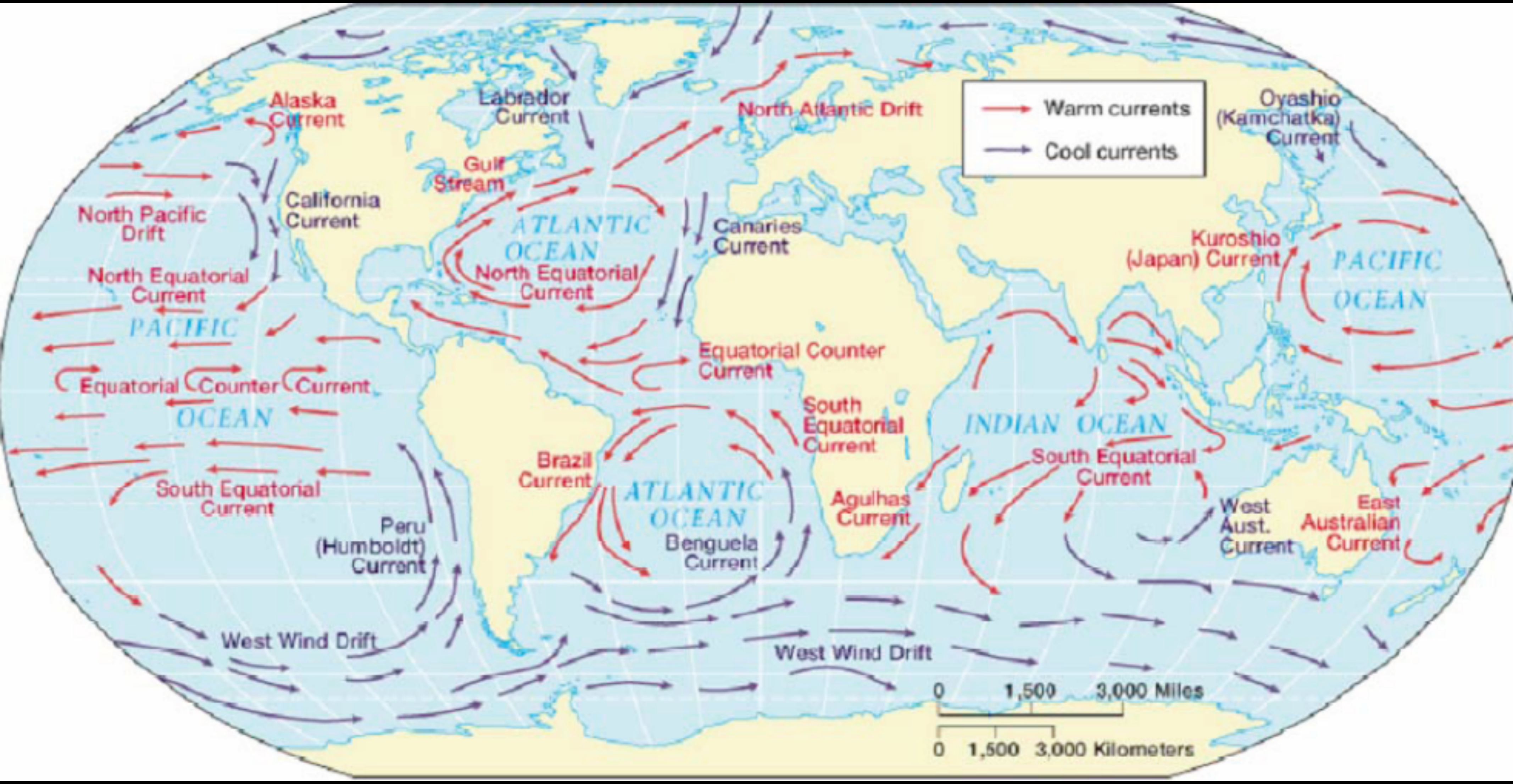
An important effect of the fluid envelope: unbalance between absorbed and emitted radiation



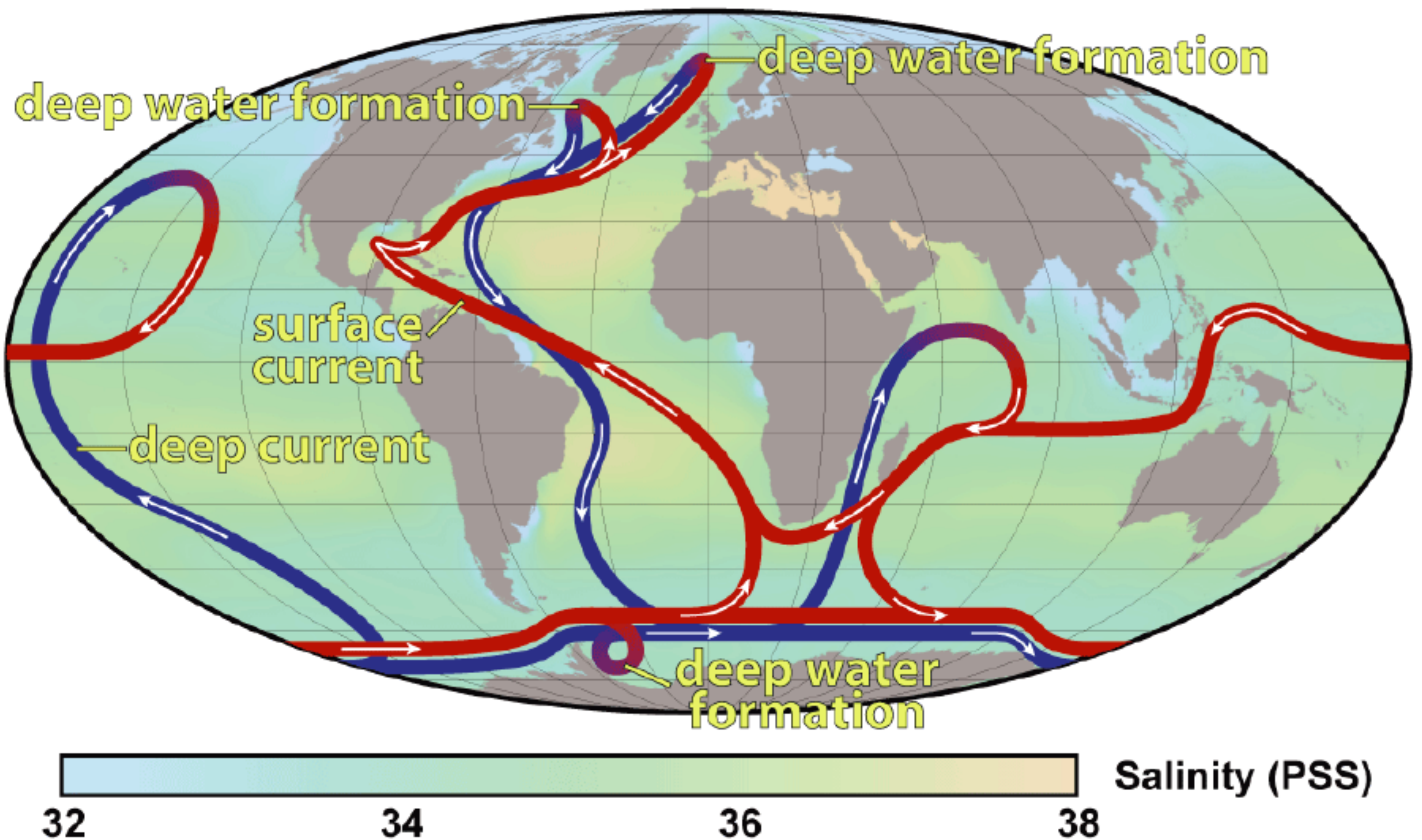




Atmospheric circulation

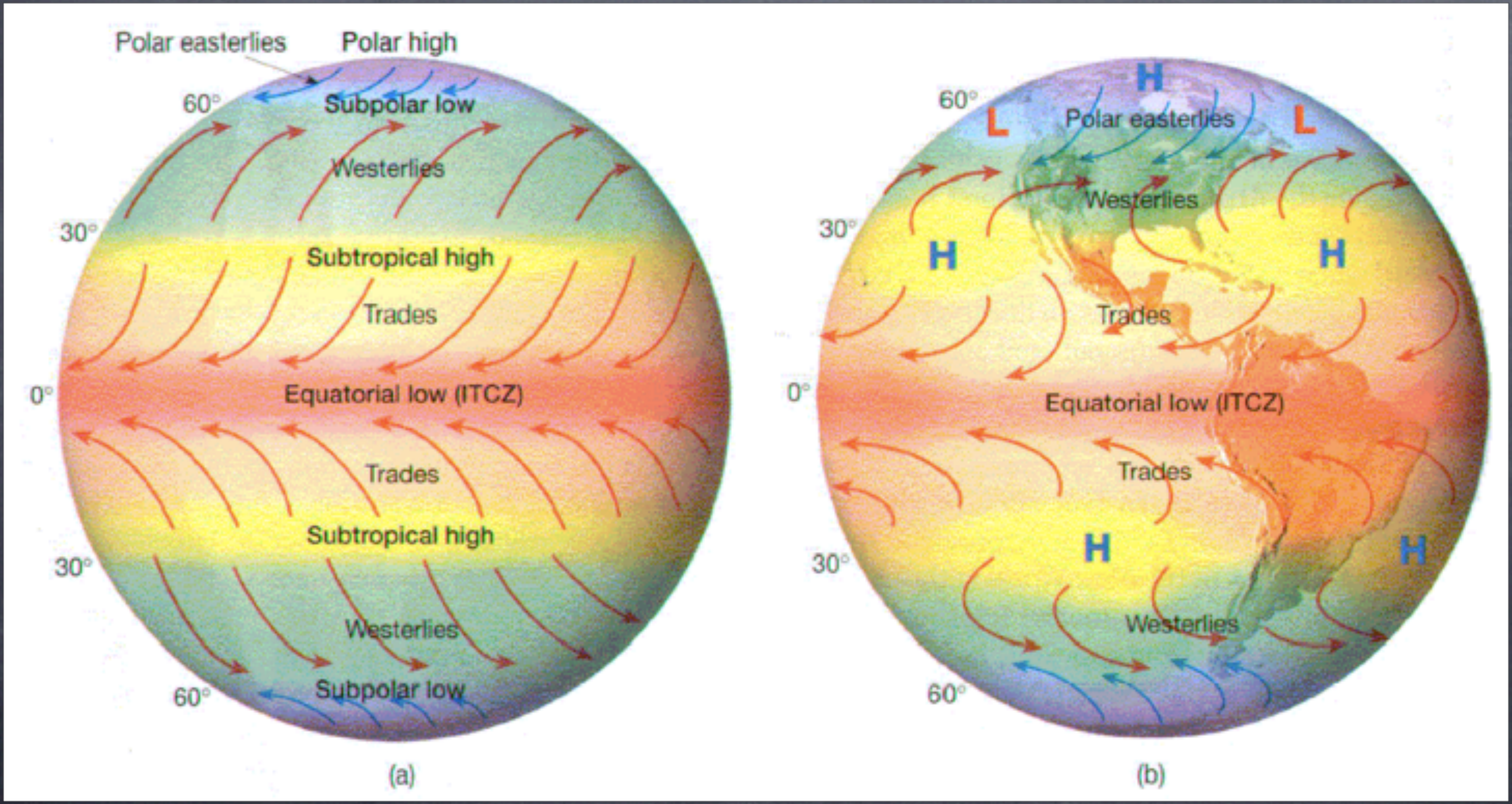


Wind-driven ocean circulation

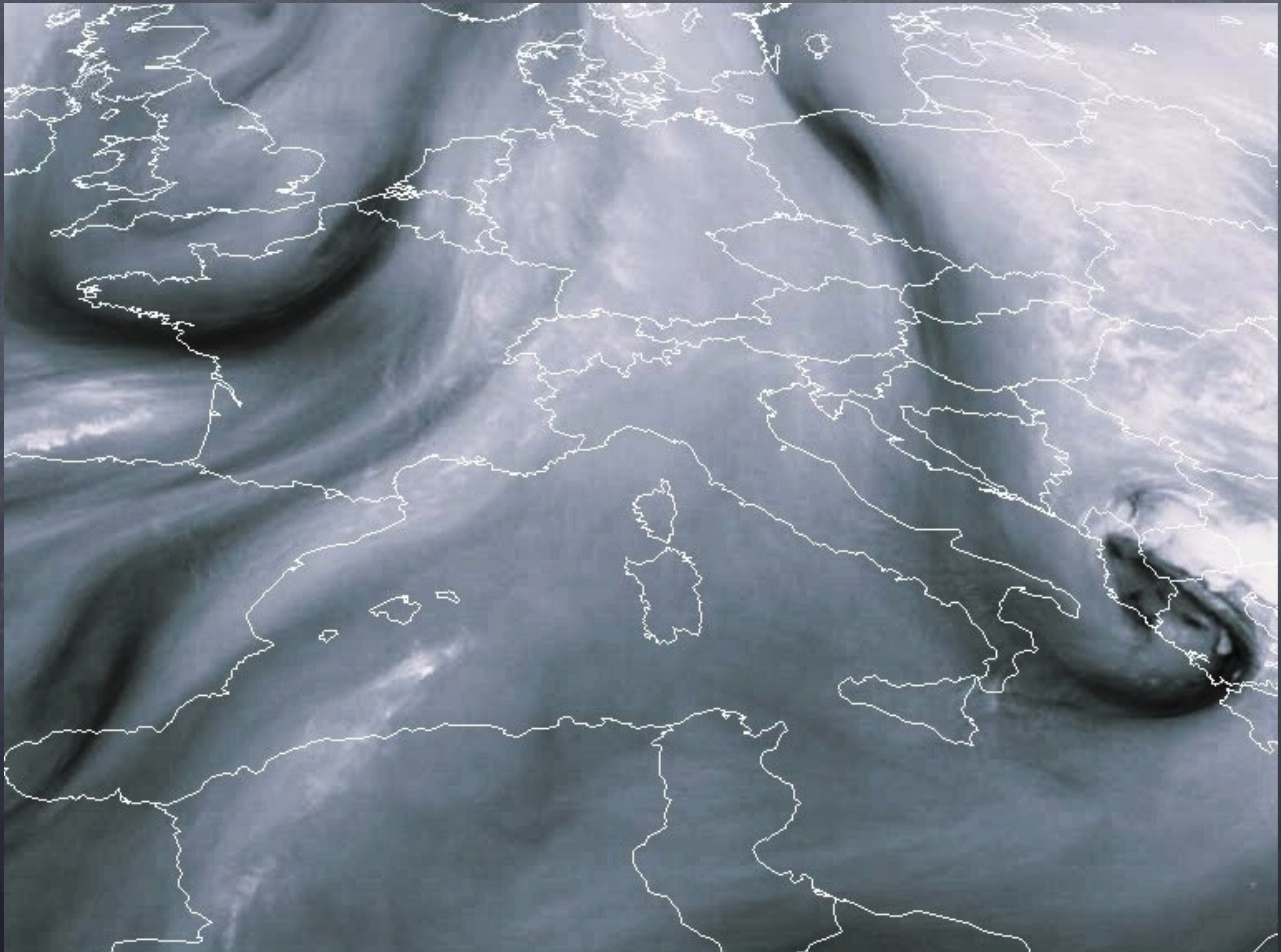


Thermohaline circulation

Atmospheric dynamics at midlatitudes



Atmospheric dynamics at midlatitudes



Atmospheric dynamics at midlatitudes: the Lorenz model of 1983

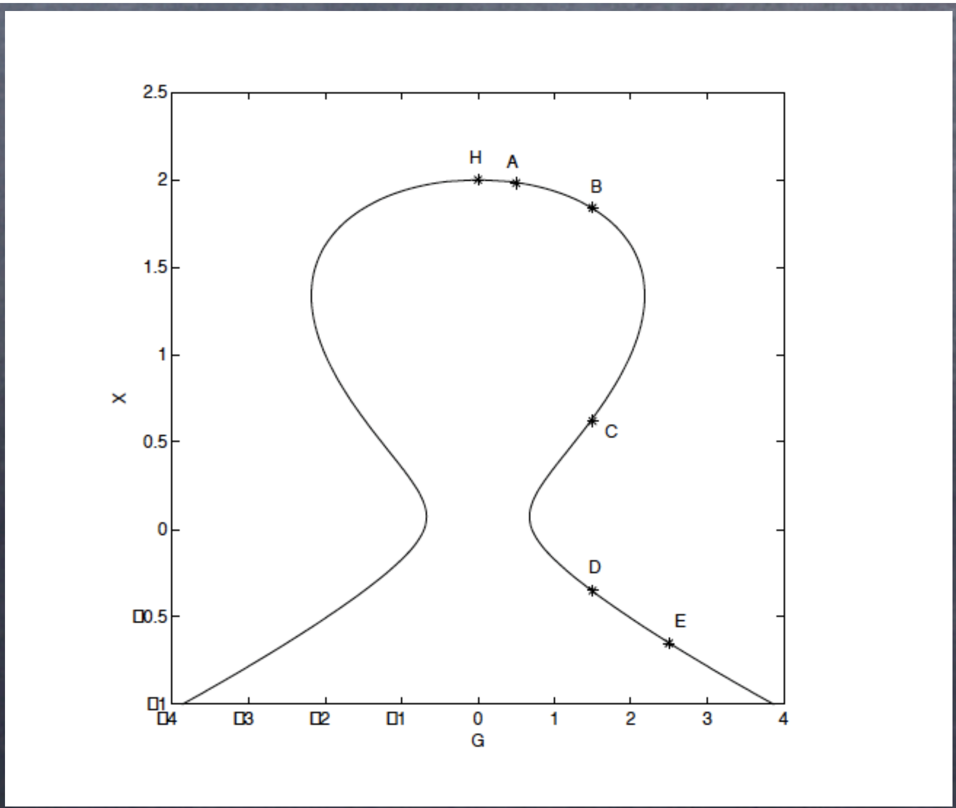
$$\frac{dX}{dt} = -Y^2 - Z^2 - aX + aF$$
$$\frac{dY}{dt} = XY - bXZ - Y + G$$
$$\frac{dZ}{dt} = bXY + XZ - Z.$$

fixed points of the Lorenz model of 1983

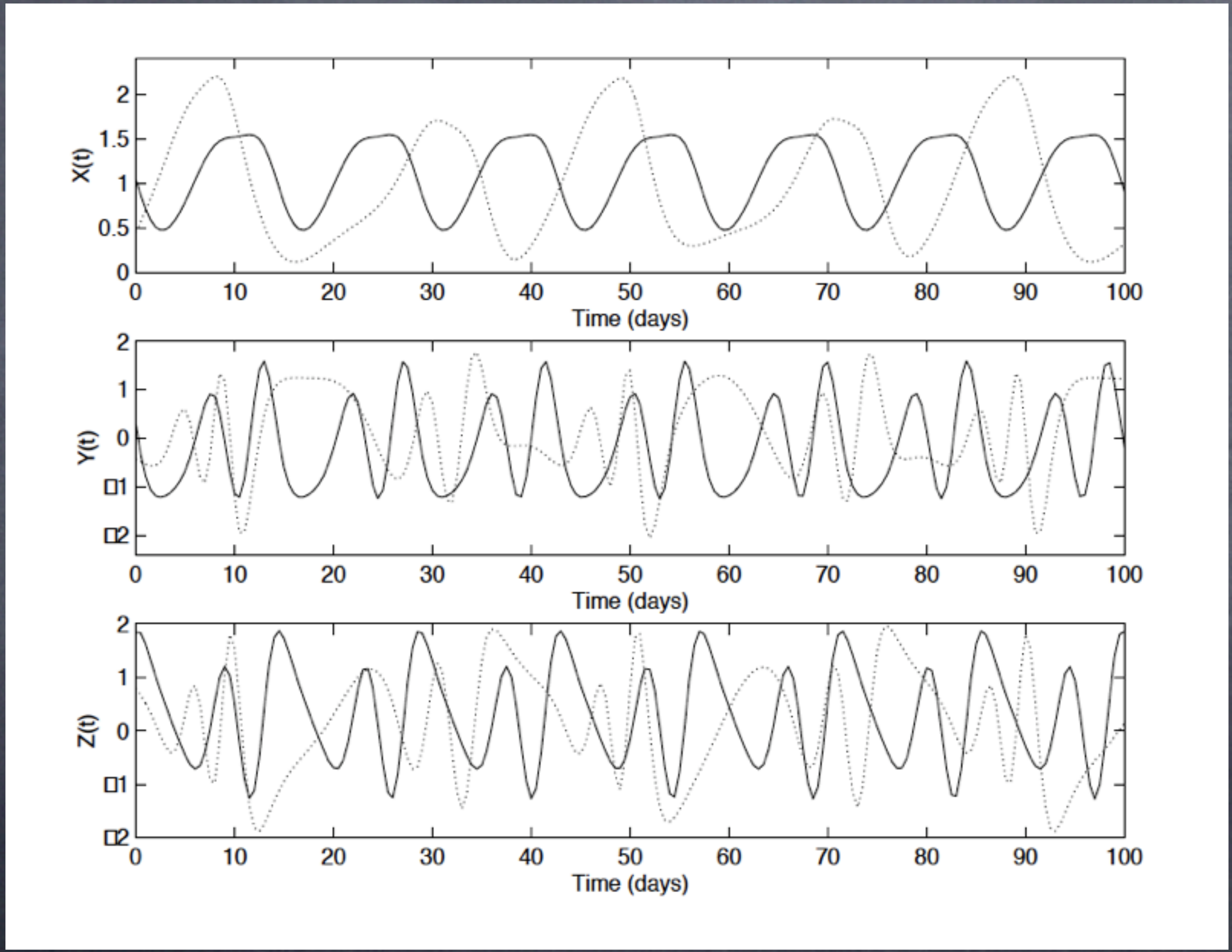
$$Y = \frac{(1-X)G}{1-2X+(1+b^2)X^2},$$

$$Z = \frac{bXG}{1-2X+(1+b^2)X^2}$$

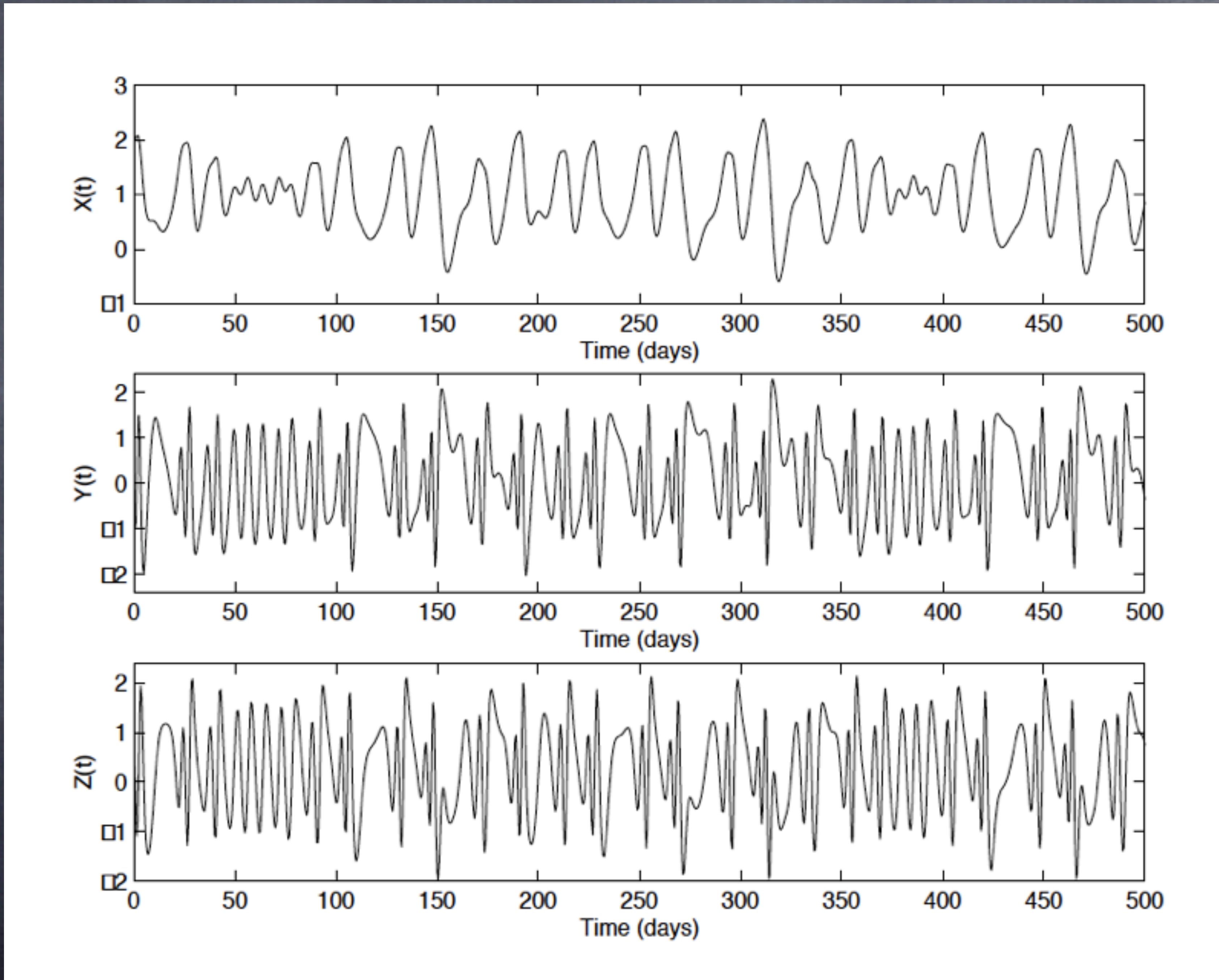
$$a(F-X)(1-2X+(1+b^2)X^2) - G^2 = 0.$$



intransitivity of the Lorenz model of 1983



chaotic dynamics of the Lorenz model of 1983



One-dimensional Energy Balance Models:

Zonal averages

Diffusive meridional transport

$$C\frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left[D\left(1-x^2\right)\frac{\partial T}{\partial x}\right] + I = S\left(1-A\right)$$

$$x = \sin\varphi.$$

with G. Vladilo, G. Murante, L. Silva

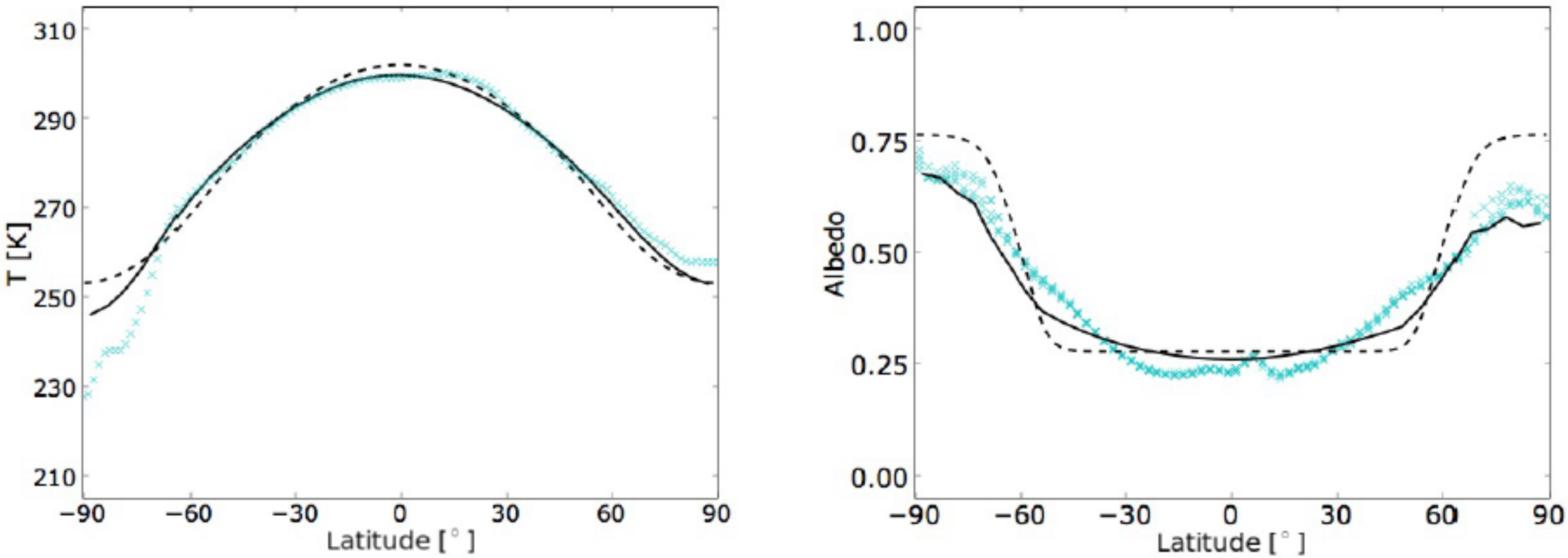


Fig. 1.— Comparison of experimental data and model predictions of the Earth latitude profiles of mean annual temperature (left panel) and mean annual albedo (right panel). Crosses: average ERA Interim temperatures in the period 1979-2010 (left panel); ERBE short-wavelength albedo in the years 1985-1989 (Pierrehumbert 2010). Solid line: our model. Dashed line: model by SMS08.

G. Vladilo, G. Murante, L. Silva, AP, G. Ferri, G. Ragazzini, *sub judice* 2013

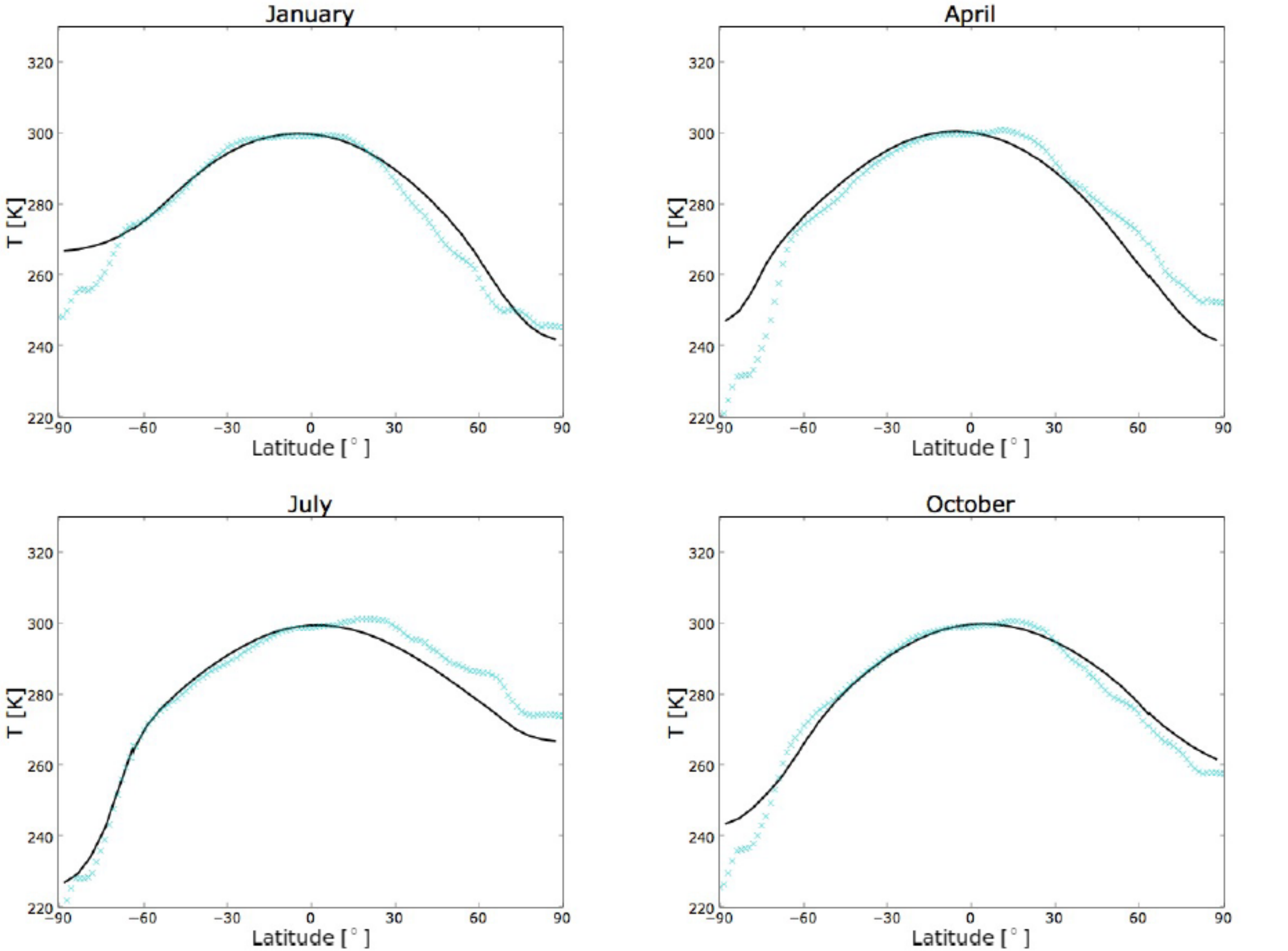


Fig. 2.— Comparison of experimental data and model predictions of the Earth latitude profiles of mean monthly temperature for January, April, July and October. Crosses: average ERA Interim data for the same months collected in the period 1979-2010. Solid line: predictions of our EBM.

What can you do with 1D EBM

Planetary habitability

Long (and ancient) paleo simulations

Explore exotic situations

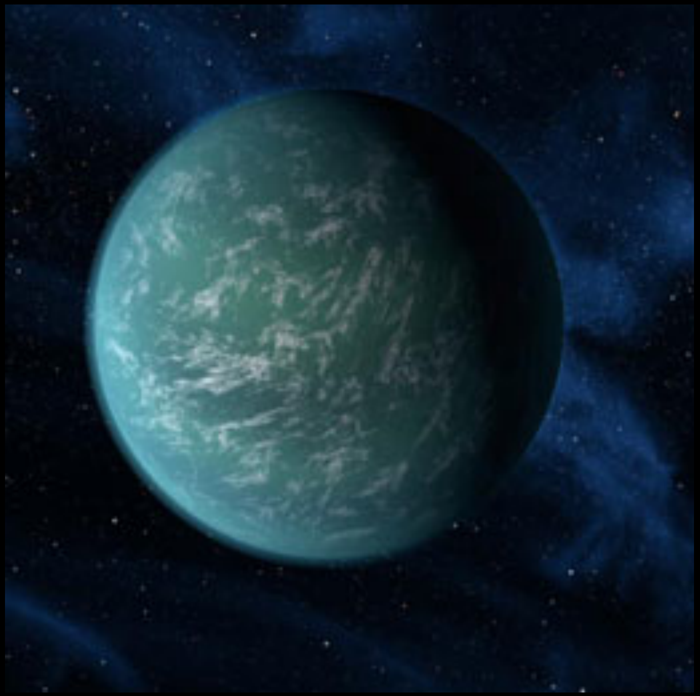
Learn a lot of Physics

Appendix

1D EBMs and planetary abitability

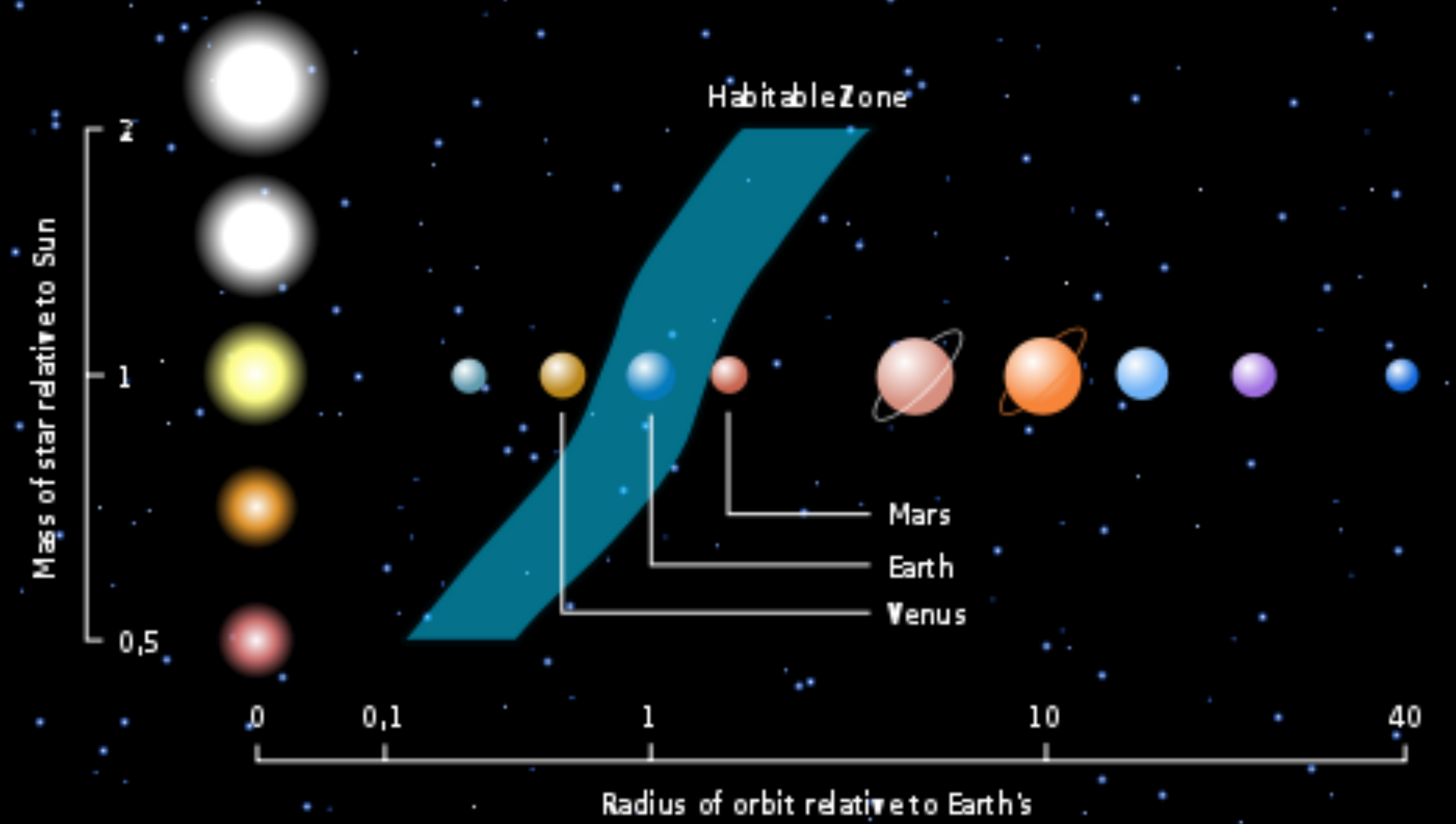
with G. Vladilo, G. Murante, L. Silva

Planetary abitability



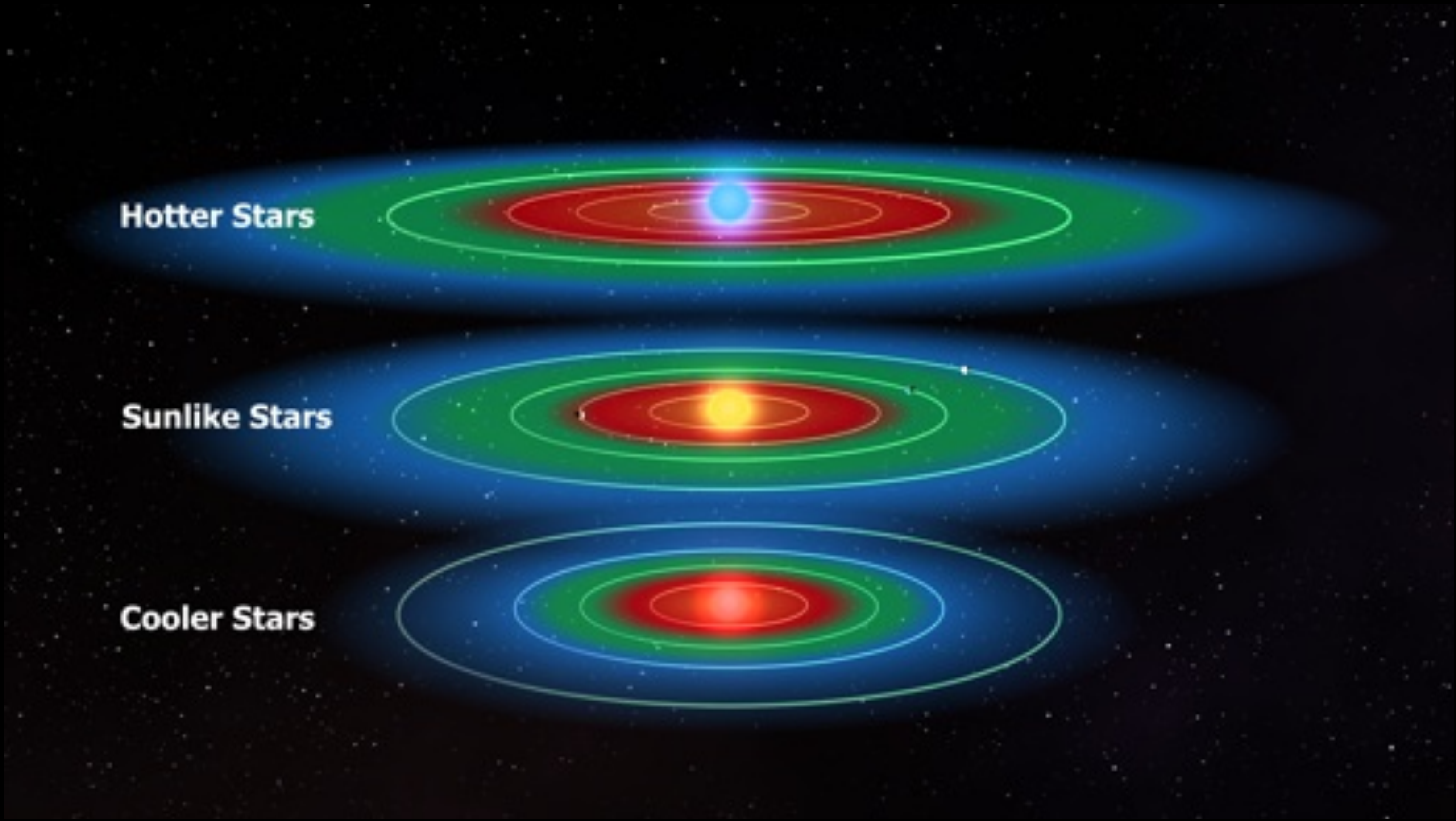
Kepler mission, NASA

Planetary abitability



https://www.e-education.psu.edu/astro801/content/l12_p4.html

Planetary abitability



https://www.e-education.psu.edu/astro801/content/l12_p4.html

Habitability

$$H(\varphi,t) = \begin{cases} 1 & \text{if } T_{\text{ice}}(p) \le T(\varphi,t) \le T_{\text{vapor}}(p) \\ 0 & \text{otherwise} \end{cases} .$$

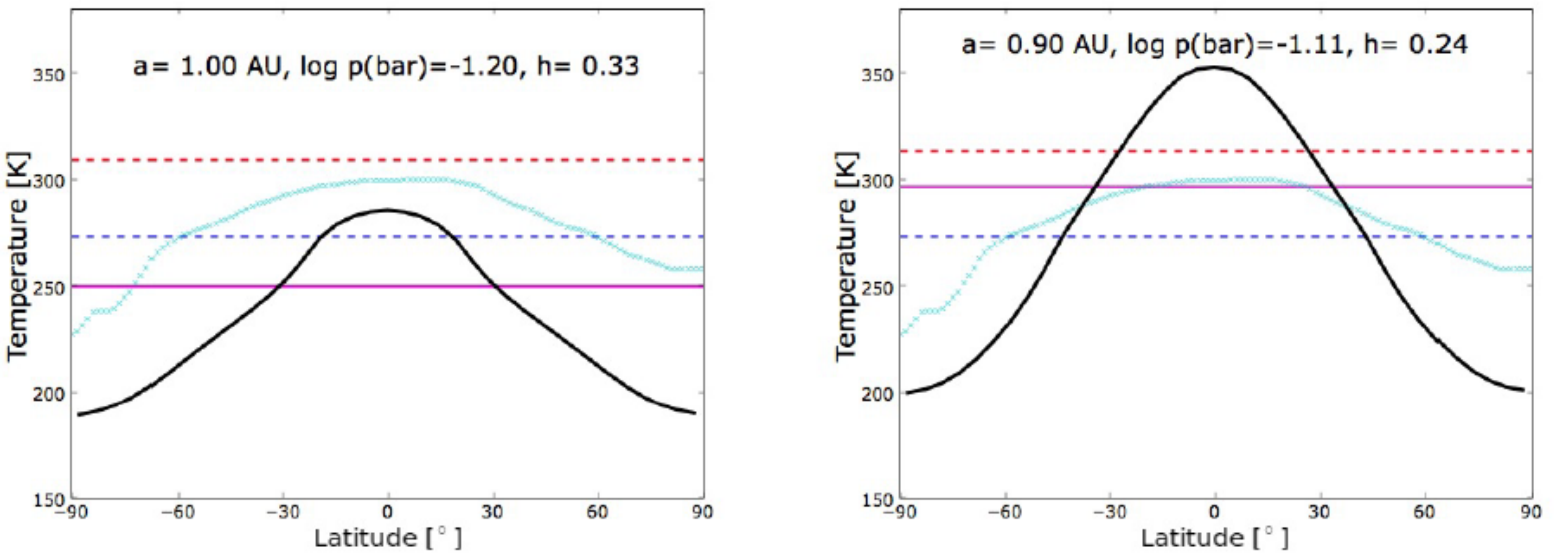
$$f_{\text{time}}(\varphi) = \frac{\int_0^P dt\ H(\varphi,t)}{P}$$

$$f_{\text{area}}(t) = \frac{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\varphi\ [H(\varphi,t)\cos\varphi]}{2}$$

$$h = \frac{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\varphi \int_0^P dt\ [H(\varphi,t)\cos\varphi]}{2P} .$$

mean fraction of planet surface that is habitable during the orbital period.

What habitability?



$$C\frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left[D\left(1-x^2\right)\frac{\partial T}{\partial x}\right] + I = S\left(1-A\right)$$

$$C = f_l[(1-f_i)\,C_l + f_i\,C_i] + f_o[(1-f_i)C_o + f_i\,C_i]$$

$$\begin{aligned} a_s = f_o\Big\{&(1-f_i)\Big[a_o(1-f_{cw}) + a_c f_{cw}\Big] + \\ &+ f_i\Big[a_{io}(1-f_{ci}) + a_c f_{ci}\Big]\Big\} + \\ &+ f_l\Big\{(1-f_i)\Big[a_l(1-f_{cl}) + a_c f_{cl}\Big] + \\ &+ f_i\Big[a_{il}(1-f_{ci}) + a_c f_{ci}\Big]\Big\}\,. \end{aligned}$$

$$\begin{aligned} a_o = &\frac{0.026}{(1.1\,\mu^{1.7} + 0.065)} + \\ &+ 0.15(\mu-0.1)\,(\mu-0.5)\,(\mu-1.0) \end{aligned}$$

$$\mu = \cos Z_\star$$

$$C\frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left[D\left(1-x^2\right)\frac{\partial T}{\partial x}\right] + I = S\left(1-A\right)$$

Outgoing infrared radiation:

From empirical formulas or using the results of a radiative-convective model

Model	IR Cooling Function	Albedo Function
1[a]	$I_1[T] = \frac{\sigma T^4}{1+(3/4)\tau_{\mathrm{IR}}^0}$	$A_1[T] = 0.5 - 0.2\tanh[\frac{(T-268\mathrm{K})}{5\mathrm{K}}]$
2[b]	$I_2[T] = \frac{\sigma T^4}{1+(3/4)\tau_{\mathrm{IR}}[T]}$	$A_2[T] = 0.525 - 0.245\tanh[\frac{(T-268\mathrm{K})}{5\mathrm{K}}]$
3[c]	$I_3[T] = A + BT$	$A_3[T] = 0.475 - 0.225\tanh[\frac{(T-268\mathrm{K})}{5\mathrm{K}}]$

$$C\frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left[D\left(1-x^2\right)\frac{\partial T}{\partial x}\right] + I = S\left(1-A\right)$$

Diffusion coefficient

$$\left(\frac{D}{D_\circ}\right) = \zeta_\varepsilon(\varphi, t)\left(\frac{p}{p_\circ}\right)\left(\frac{c_\mathrm{p}}{c_{p,\circ}}\right)\left(\frac{m}{m_\circ}\right)^{-2}\left(\frac{\Omega}{\Omega_\circ}\right)^{-2}$$

Incoming solar radiation

$$\cos Z_\star = \sin\varphi \, \sin\delta_\star + \cos\varphi \, \cos\delta_\star \, \cos t_\star$$

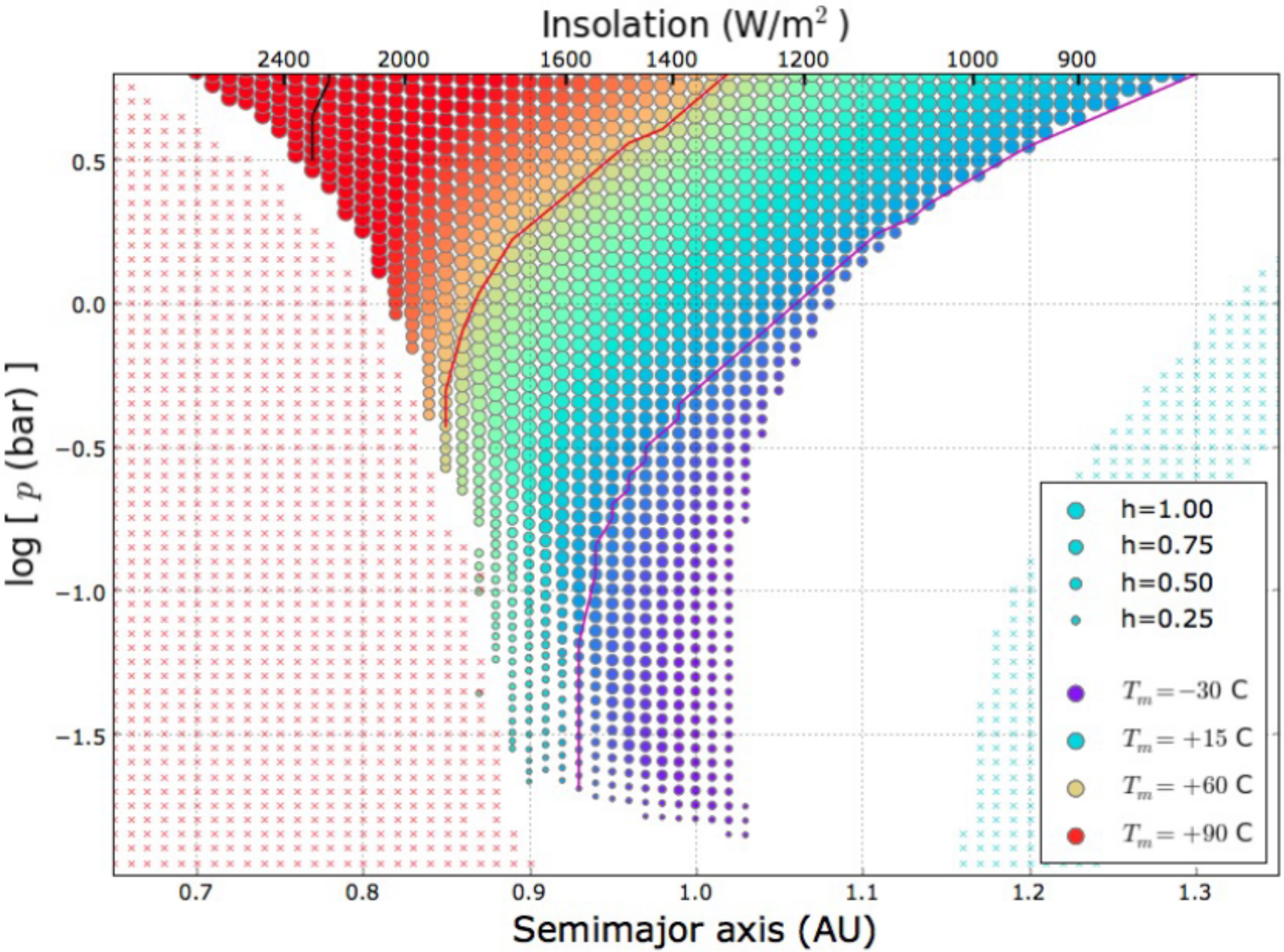
$\mu = \cos Z_\star$

$Z_\star$ the stellar zenith angle

$$\overline{\mu} = \sin\varphi \, \sin\delta_\star + \cos\varphi \, \cos\delta_\star \, \frac{\sin H}{H}$$

$$S = \frac{q(r)}{\pi} \left(H \sin\varphi \, \sin\delta_\star + \cos\varphi \, \cos\delta_\star \, \sin H \right)$$

$$q(r) = \frac{L_\star}{4\pi r^2}$$



Insolation (W/m^2)
2400
2000
1600
1400
1200
1000
900
log [p (bar)]
0.5
0.0
−0.5
−1.0
−1.5
0.7
0.8
0.9
1.0
1.1
1.2
1.3
Semimajor axis (AU)
h=1.00
h=0.75
h=0.50
h=0.25
$T_m = -30$ C
$T_m = +15$ C
$T_m = +60$ C
$T_m = +90$ C

Fig. 4.— Circumstellar habitable zone of planets with Earth-like atmospheres and different levels of surface pressure obtained with our EBM climate simulations. Abscissae: semi-major axis, a (bottom axis), or insolation $q = L_\star/(4\,\pi\,a^2)$ (top axis). Ordinates: logarithm of the total surface pressure, p. The circles indicate solutions with mean global annual habitability $h > 0.1$. The area of the symbols is proportional h; the colors are coded according to the mean annual global surface temperature, T_m. The size and color scales are shown in the legend. Solid lines: curve of constant mean temperature $T_m = 0\,°C$, $60\,°C$ and $120\,°C$ (from right to left, respectively). Pixels in which the simulation has been forced to exit are marked with crosses (see Section 3.1). Results above the red line are tentative (see Section 3.3). Adopted model parameters are listed in Tables 2 and 3.

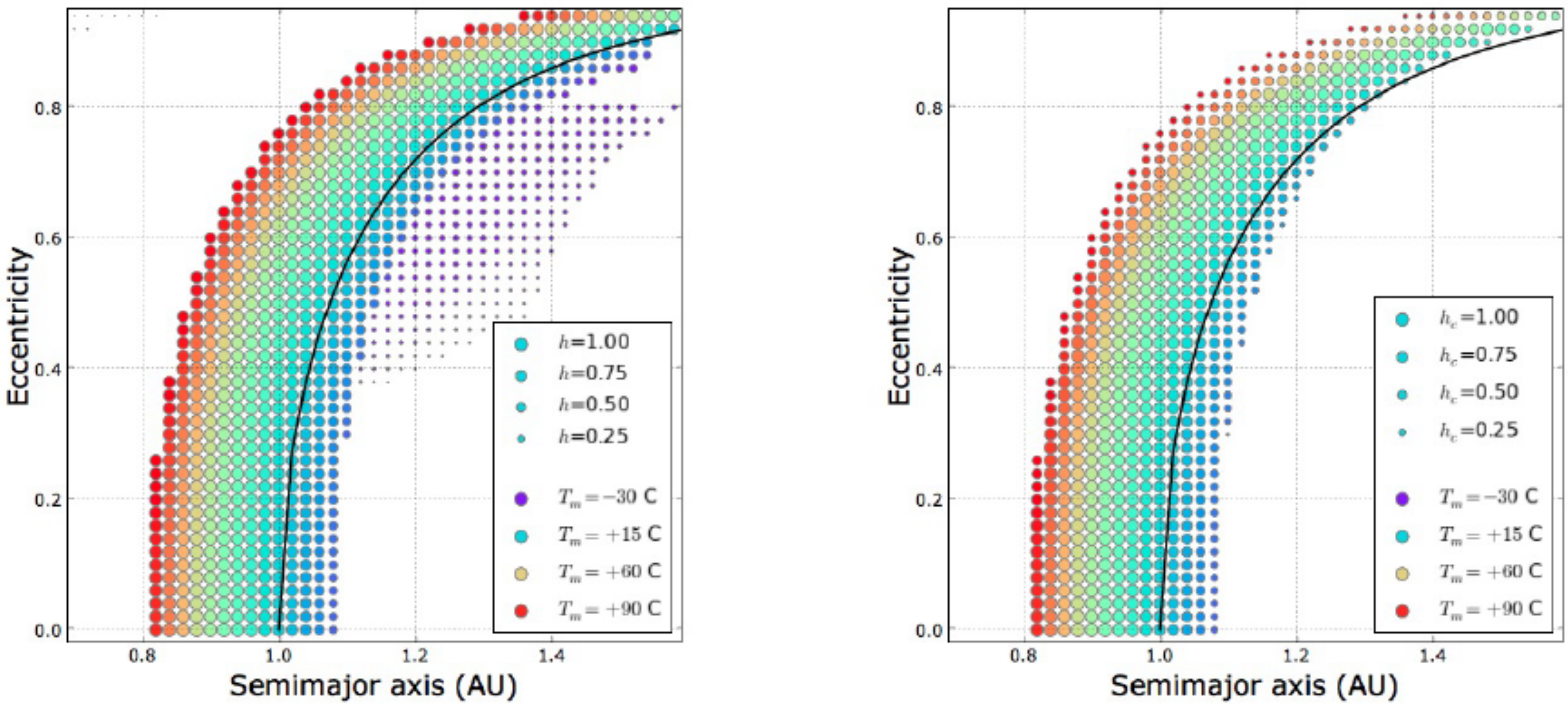


Fig. 7.— Maps of mean temperature and habitability of an Earth-like planet in the plane of the semi-major axis and eccentricity. Left panel: habitability h; right panel: continuous habitability h_c (see Section 3.2). The area of the circles is proportional to the mean fractional habitability; the color varies according to the mean annual global surface temperature, T_m. The size and color scales are shown in the legend. Adopted model parameters are listed in Tables 2 and 3, with the exception of the eccentricity that has been varied as shown in the figure. Solid curve: line of equal mean annual flux $<q> = q_0$ estimated from Eq. (10).

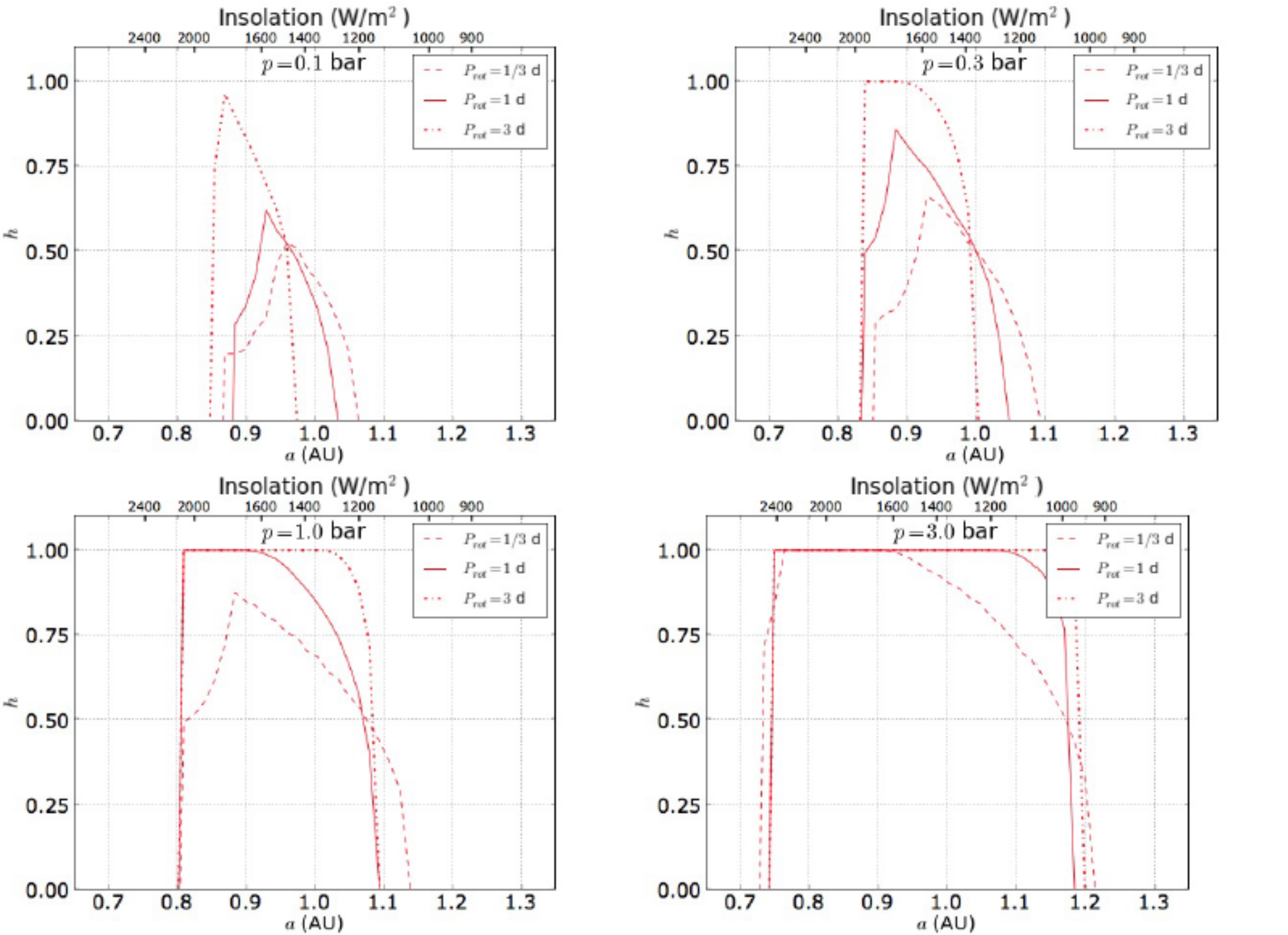


Fig. 9.— Fractional habitability, h, as a function of semi-major axis, a, for planets with rotation periods $P_{\rm rot}$ = 1/3 d, 1 d, and 3 d. Each panel shows the results obtained at a constant pressure p. The other parameters of the simulations are listed in Tables 2 and 3.

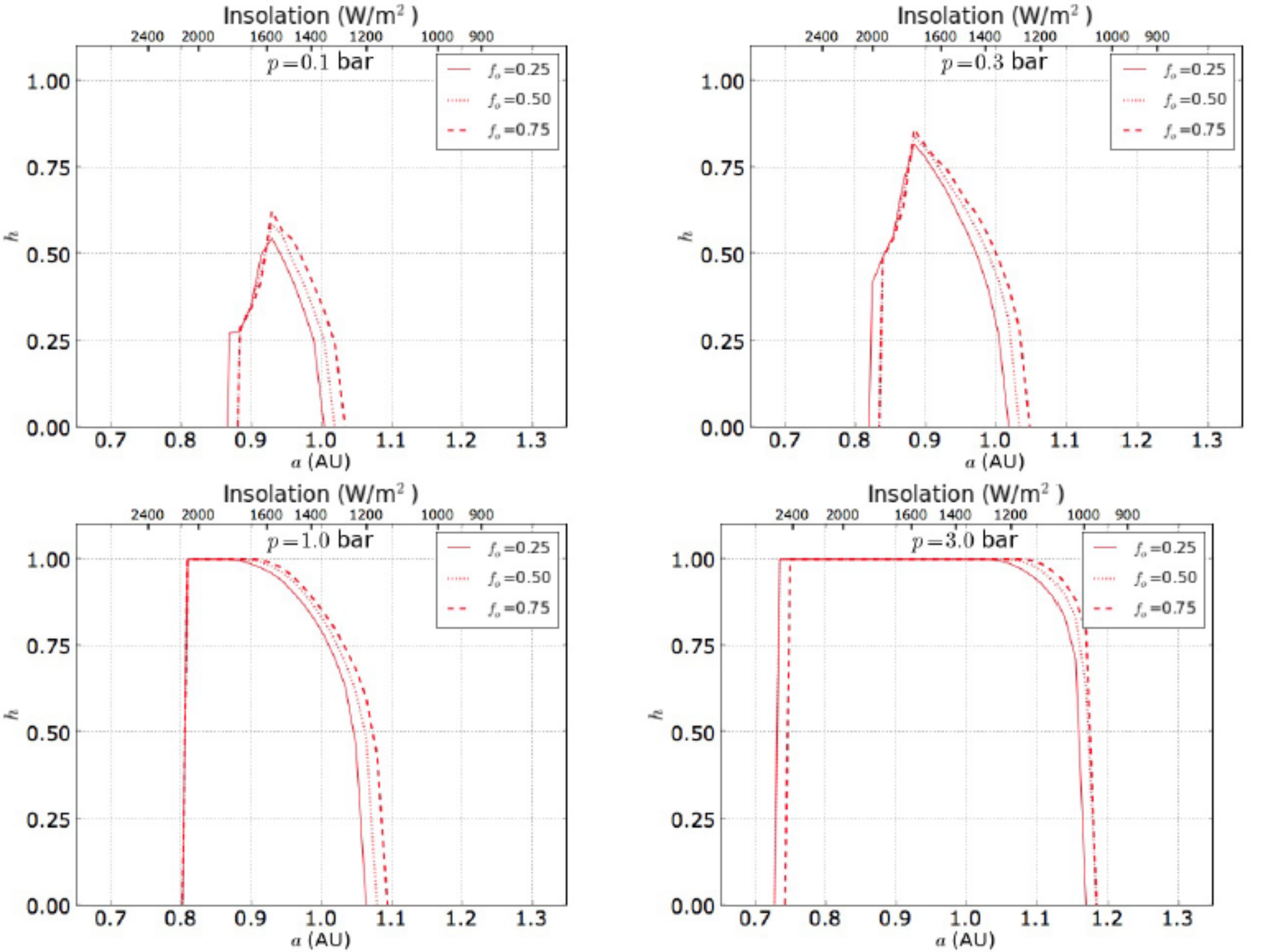


Fig. 11.— Fractional habitability, h, as a function of semi-major axis, a, for planets with ocean fractions $f_o = 0.25$, 0.50, and 0.75. Each panel shows the results obtained at a constant pressure p. The other parameters of the simulations are listed in Tables 2 and 3.

Or, study ancient climate
(Archean, Adean, Snowball Earth…)

Add ice sheets
Add other components…