A tömeg eredete a látható világegyetemben

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MTA-ELTE ”Lendület” rácstérelmélet kutatócsoport

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Outline

Motivation

Composition of the Universe

- 68% dark energy
- 27% dark matter
- 5% ordinary matter

Dark energy and dark matter are yet unknown

Ordinary matter (visible Universe):

- Standard Model particles and their bound states
- mostly (in mass) atoms and nuclei
- 99.9% of the mass comes from protons & neutrons
- proton, neutron:
- not elementary particles, built from quarks
- their masses are non-trivial, a consequence of the strong, electromagnetic and weak interactions

Big bang nucleosynthesys and nuclei chart

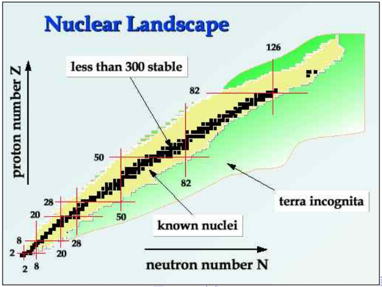
$$\Delta m_N = m_n - m_p = 0.0014 m_p$$

Δm_N too small $\rightarrow$ inverse β decay leaving predominantly neutrons

$\Delta m_N \approx 0.05\%$ would already lead to much more *He* and much less *H* $\rightarrow$ stars would not have ignited as they did

$\Delta m_N > 0.14\%$ $\rightarrow$ much faster beta decay, less neutrons after BBN burning of *H* in stars and synthesis of heavy elements difficult

The whole nuclei chart is based on precise value of Δm_N



The 3 interactions in the Standard Model

Strong (QCD)

quarks' internal degree of freedom: color

SU(3) symmetry in color space

coupling ≈ 1

Electromagnetic

among charged particles

U(1) symmetry after the electroweak breaking

coupling: $\approx 1/137$

Weak interaction

original $SU(2) \times U(1)$ symmetry spontaneously broken by the Higgs

no exact symmetry at low energies

approximate isospin symmetry is a remnant

coupling: $\approx 10^{-5}$

+1: gravity

coupling: $\approx 10^{-40}$

Origin of mass

Masses of elementary particles

Higgs mechanism:

Higgs-fermion Yukawa couplings (λ_i) lead to masses

$m_i = \lambda_i v/\sqrt{2}$

This is the contribution of the weak interaction

$m_e = 0.511 \text{ MeV}$

Quark masses are not uniquely defined. E.g. in $\overline{MS}(2\text{GeV})$ scheme:

$m_u = 2.2(1) \text{ MeV}$

$m_d = 4.8(2) \text{ MeV}$

Mass of the *H* atom: $m_H = 940 \text{ MeV}$

Where does the rest come from?

Mass of the proton and neutron

Proton and neutron are not elementary:

$p = uud \quad n = udd$

Besides the quark masses all energy (strong and EM) contributes

Strong interaction

quarks and gluons confined into the hadrons have
huge kinetic and interaction energies

This is the dominant contribution (99%)

Electromagnetic interaction

Charged particles have a static electric field which stores energy

Weak interaction

responsible for the quark masses through the Higgs mechanism

negligible beyond that

Determination of the proton/neutron masses

Approximate solution

Isospin symmetry:

All properties of the u, d quarks are identical

their charges are neglected

good approximate symmetry but leads to identical m_n and m_p

Only QCD and the quark masses contribute

More detailed analysis

isospin symmetry is not exact

$m_u \neq m_d$ and $Q_u = 2/3 \;\; Q_d = -1/3$

QCD + QED + quark masses contribute

estimate:

quark masses: $\Delta m_N \approx 2.6 \text{ MeV}$

electrostatic energy of the proton: $\Delta m_N \approx -0.9 \text{ MeV}$

Quantum Chromodynamics (QCD)

QCD: Currently the best known theory to describe the strong interaction.

SU(3) gauge theory with fermions in fundamental representation.

Fundamental degrees of freedom:

- gluons: $A_\mu^a, \quad a = 1, \dots, 8$
- quarks: $\psi, \quad$ 3(color) $\times$ 4(spin) $\times$ 6(flavor) components

$$\mathcal{L}_{QCD} = \underbrace{-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}_{\text{pure gauge part}} + \underbrace{\overline{\psi}(iD_\mu \gamma^\mu - m)\psi}_{\text{fermionic part}},$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$ field strength

$$D_\mu = \partial_\mu + g A_\mu^a \frac{\lambda^a}{2i}$$ covariant derivative $\longrightarrow$ gives quark–gluon interaction

Quantum Chromodynamics (2)

$\mathcal{L}_{\text{QCD}}$ is invariant under local gauge transformations:

$$A'_\mu(x) = G(x)A_\mu(x)G(x)^\dagger - \frac{i}{g}\left(\partial_\mu G(x)\right) G(x)^\dagger$$
$$\psi'(x) = G(x)\psi(x)$$
$$\overline{\psi}'(x) = \overline{\psi}(x)G^\dagger(x)$$

Only gauge invariant quantities are physical.

Properties of QCD:

- Asymptotic freedom:
  Coupling constant $g \to 0$ when energy scale $\mu \to \infty$.
  $\Longrightarrow$ Perturbation theory can be used at high energies.
- Confinement:
  Coupling constant is large at low energies.
  $\Longrightarrow$ Nonperturbative methods are required.

Quantum Chromodynamics (3)

Quantization using Feynman path integral:

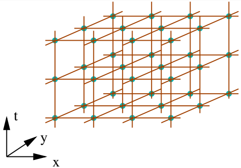
$$\langle 0|\, T[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)]\,|0\rangle = \frac{\int [\mathrm{d}\psi]\,[\mathrm{d}\overline{\psi}]\,[\mathrm{d}A_\mu]\,\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\, e^{iS[\psi,\overline{\psi},A_\mu]}}{\int [\mathrm{d}\psi]\,[\mathrm{d}\overline{\psi}]\,[\mathrm{d}A_\mu]\, e^{iS[\psi,\overline{\psi},A_\mu]}}$$

e^{iS} oscillates $\longrightarrow$ hard to evaluate integrals.
Wick rotation: $t \to -it$ analytic continuation to Euclidean spacetime.
$\Longrightarrow \quad e^{iS} \longrightarrow e^{-S_E}$, where

$$S_E = \int \mathrm{d}^4x\, \mathcal{L}_E = \int \mathrm{d}^4x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}(D_\mu \gamma^\mu + m)\psi\right]$$

positive definite Euclidean action.

Lattice regularization



take a finite spacetime volume (TV) and discretize with a lattice
gluons live on links, quarks live on sites
path integral $\rightarrow$ finite ($\mathcal{O}(10^9)$) number of integrals
quarks integrated analytically: $S \rightarrow S_{eff}$
numerical Monte Carlo integration possible for the gauge fields

Importance sampling

generate configurations with $\exp(-S_{eff})$ weight

Particle masses

two point functions: $\langle O(0)O(t)\rangle \xrightarrow[t\to\infty]{} \mathrm{const} \cdot \exp(-\mathrm{mt})$

High performance computing required

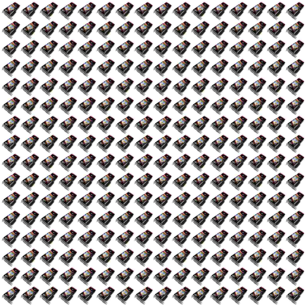
Juqueen system in Jülich

– 458752 cores

– 6 Pflop/s peak performance

– 2 Pflop/s sustained performance for Monte-Carlo codes

GPU cluster at the Eötvös University



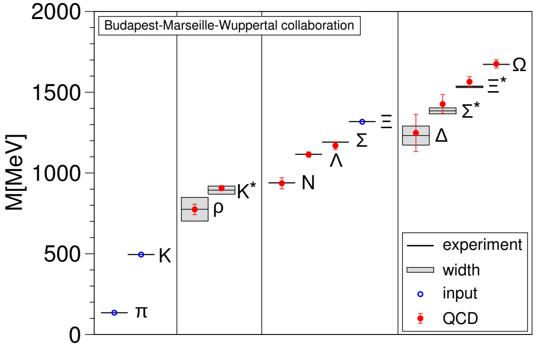
GPU cluster at ELTE

– 387072 cores

– 1.1 Pflop/s peak performance

– 78 Tflop/s sustained performance

Isospin symmetric case



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masses extracted from various two-point functions

correlation length → hadron masses

perfect agreement with experiments

The challenge of computing $m_n - m_p$

Unprecedented precision is required

$\Delta m_N / m_N = 0.14\%$ $\rightarrow$ sub-permil precision is needed to get a high significance on Δm_N

$m_u \neq m_d$ $\rightarrow$ 1+1+1+1 flavor lattice calculations are needed $\rightarrow$ algorithmic challenge
(Previous QCD calculations were typically 2+1 or 2+1+1 flavors)

Inclusion of QED: no mass gap
$\rightarrow$ power-like finite volume corrections expected
$\rightarrow$ long range photon field may cause large autocorrelations

Overview of full QED-QCD simulations

- Fully dynamical lattice calculation of $SU(3) \times U(1)$ gauge theory with non-degenerate u,d,s,c quarks
- Interpolation/extrapolation to physical point (continuum limit, physical quark masses)
- Treat power-like finite volume corrections analytically/numerically
- Potential precision: $\mathcal{O}(\frac{1}{N_c m_b^2}, \alpha^2) \sim 10^{-4}$
- Calculate mass splittings of N, Σ, Ξ as well as charmed hadrons, D, Ξ_{cc} and Coleman-Glashow relation
- Reaching the desired accuracy via a blind analysis (hard)
- Determine QCD/QED contributions of splittings

Zero mode subtraction

The absence of a mass gap may cause divergences at finite volume
perturbative momentum sums $\rightarrow$ $1/k^2$ factors $\rightarrow$
zero mode problematic

Removing a finite number of modes does not change $V \rightarrow \infty$ physics

Advantages of zero mode removal:
$\rightarrow$ analytic calculation of finite V corrections is possible
$\rightarrow$ algorithmic speedup

Many possibilities, we use:

(Hayakawa, Uno) all spatial zero modes: $\sum_{\vec{x}} A_{\mu,x_0,\vec{x}} = 0 \quad \forall \mu, x_0$

QED in a finite volume

Calculate 1 loop self energy of charged particles in finite and infinite volumes

For a point-like particle

$$\Delta\Sigma(p,L) = \left[\sum_k - \int \frac{d^4k}{(2\pi)^4} \right] \sigma(k,p)$$

zero mode removal: $\sum_k \equiv \frac{1}{TL^3} \sum_{k_0} \sum_{\vec{k}\neq 0}$

Finite V correction to the pole mass can be calculated

Result for a spin half particle:

$$m(L) \underset{L\to+\infty}{\sim} m \left\{ 1 - q^2\alpha \left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3} \right] \right\}$$

with $\kappa = 2.837297(1)$

Composite particles

In QCD charged hadrons are not point-like
previous results have to be extended

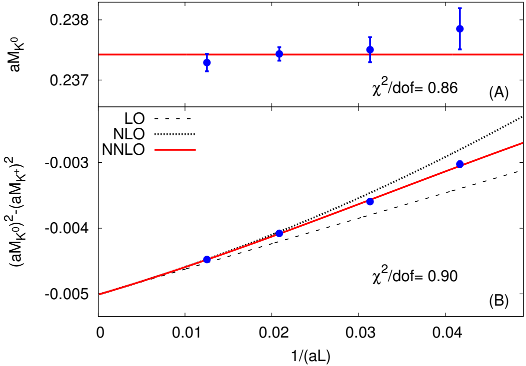
QED Ward identities → first two orders universal:

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1 - q^2\alpha\frac{\kappa}{2mL}\left[1+\frac{2}{mL}\right]+\mathcal{O}(\frac{\alpha}{L^3})\right\}$$

for scalars and spin half fermions
Form factors (e.g. charge radius) enter at $\mathcal{O}(\frac{\alpha}{L^3})$ level.

Strategy:
include analytic corrections for the two universal orders
fit coefficient of $1/L^3$
$1/L^3$ in many cases negligible, only significant for mesons

Finite V dependence of the kaon mass



Neutral kaon shows no volume dependence
Volume dependence of the K splitting is perfectly described
$1/L^3$ order is significant

Determining the isospin splittings

two sources of isospin violation: electromagnetism & $m_u \neq m_d$

we work at various elecromagnetic couplings and quark masses
renormalized coupling defined at hadronic scales

$\Rightarrow$ only linear term in α

$\delta m = m_d - m_u$ is very small $\Rightarrow$ linear term is also enough for δm

$\Rightarrow \Delta M_X = F_X(M_{\pi^+}, M_{K^0}, M_{D^0}, L, a) \cdot \alpha + G_X(M_{\pi^+}, M_{K^0}, M_{D^0}, a) \cdot \Delta M_K^2$

ΔM_K^2: QED-like L dependence: absorbed in $F_X(..., L, a)$
charged particle masses: corrected for universal finite-size effect
non-universal effects starting with $1/L^3$ are allowed in the QED part

alternative procedure: use ΔM_Σ (less precise) instead of ΔM_K^2

Separating QED and QCD effects

if α or $(m_u - m_d)$ vanishes QED or QCD parts disappear
separation: $\Delta M_X = \Delta_{\text{QED}} M_X + \Delta_{\text{QCD}} M_X$

it is sufficient to decompose the kaon mass squared difference

use $\Delta M_X = F_X(M_{\pi^+}, M_{K^0}, M_{D^0}, L, a) \cdot \alpha + G_X(M_{\pi^+}, M_{K^0}, M_{D^0}, a) \cdot \Delta M_K^2$

separation ambigous: depends on the choice of scheme for $m_u - m_d$

Suggested separation:
use Σ^+ and Σ^- baryons
they have the same charge squared and same spin
leading order: mass difference comes from quark masses only

Coleman-Glashow relation (1961)

suggestion: $\Delta_{\mathrm{CG}} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi$ is close to zero

remember the quark compositions, in which it cancels indeed:
$M(ddu) - M(uud) - M(sdd) + M(suu) + M(ssd) - M(ssu) = 0$

determine the leading order terms in the α and δm expansion

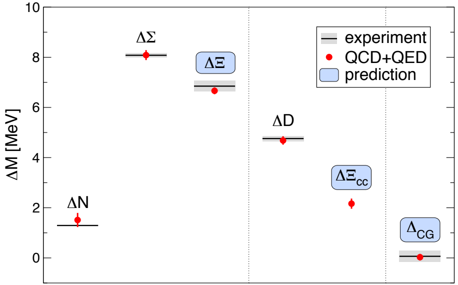
for $\alpha=0$ a complete quark exchange symmetry
$\Delta_{\mathrm{CG}} \propto (m_s - m_d)(m_s - m_u)(m_d - m_u)$

for $\alpha > 0$ remains a $d \leftrightarrow s$ symmetry, thus $\Delta_{\mathrm{CG}} \propto \alpha(m_s - m_d)$

the Coleman-Glashow relation is satisfied to high accuracy

$$\Delta_{\mathrm{CG}} = 0.00(11)(06)\ \mathrm{MeV}$$

Isospin splittings



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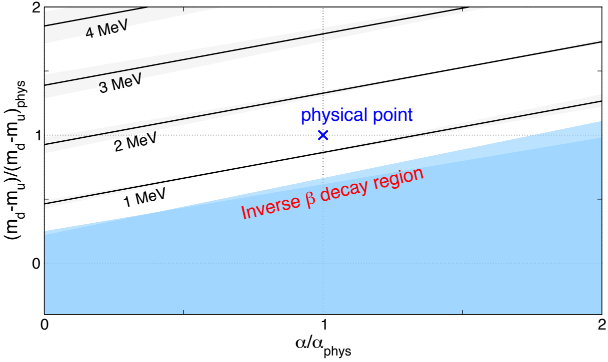
	splitting [MeV]	QCD [MeV]	QED [MeV]
ΔN=n-p	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma=\Sigma^{-}$-Σ^{+}	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi=\Xi^{-}$-Ξ^{0}	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D=D^{\pm}$-D^{0}	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc}=\Xi_{cc}^{++}$-$\Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{CG}=\Delta N-\Delta\Sigma+\Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

Dependence on fundamental parameters

How strongly does Δm_N depend on α and the quark masses?



Summary

- Mass of the visible universe: 99.9% protons and neutrons
- Their masses almost identical, but the tiny difference has cosmological consequences
- Higgs is responsible for 1% of the mass
- The rest is given mostly by QCD and a small contribution by QED
- Both masses have been determined using lattice calculations
- The mass difference is a result of two competing effects (quark masses and charges)
- Good agreement with experiments + QCD/QED contributions given separately
- 4 further splittings determined, 2 of them are predictions