Anderson átmenet a kvark-gluon plazmában

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Theory of strong interactions: QCD

Lagrangian: $\mathscr{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}[D(A) + M]\psi$

- $F_{\mu\nu}$: field strength corresponding to A — gluons
- ψ: quarks
- $D(A) = \gamma^\mu(\partial_\mu - iA_\mu)$ Dirac operator

Quantization — path integral

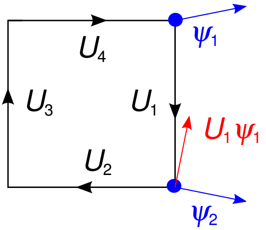
- $\mathscr{O}$ physical quantity

  $\langle \mathscr{O} \rangle \approx \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \mathscr{D}A \; \mathscr{O} \exp\left(-\int d^4x \mathscr{L}\right)$
- Integration in ∞-dimensional function space
- Mathematical definition??? How to calculate it???

QCD on the lattice

- Continuous space-time $\longrightarrow$ 4-dimensional cubic lattice
- Integration over function spaces $\longrightarrow$ finite dim. integrals
- Mathematically well-defined
- Continuum limit (how to get rid of the lattice?)
  - lattice spacing $a \to 0$
  - $\xi_{\text{lattice}} a = \frac{1}{M_{phys}}$ $\Rightarrow$ $\xi_{\text{lattice}} \to \infty$
  - tune system to critical point
  - tune a few parameters (gauge coupling, quark masses) to fix physics in $a \to 0$ limit

Lattice QCD



- $\psi_i \in \mathbb{C}^3$ (on lattice sites)
- Different basis at each lattice site
- $U_i \in SU(3)$ (complex rotation)
  - vector potential $\rightarrow$ $U \approx e^{iA}$
  - $1 \rightarrow 2$ basis transformation (ψ)
  - dynamical variables on links

Discretization

Derivative:	$\partial_\mu \psi$	$\rightarrow$	$\frac{1}{a}(\psi_2 - \psi_1)$
Covariant derivative:	$D_\mu \psi$	$\rightarrow$	$\frac{1}{a}(\psi_2 - U_1 \psi_1)$
Gluon action:	$\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$	$\rightarrow$	$-\frac{1}{g^2} \mathrm{tr}(U_1 U_2 U_3 U_4)$

Wick rotation

Analytic continuation in time $t \to -it$

- Minkowski space-time $\longrightarrow$ 4d Euclidean space
- $e^{iLt} \quad \longrightarrow \quad e^{-Ht}$
- Physical quantities = statistical averages
- Monte Carlo numerical simulation
- temporally finite box of size L_t
  $\longrightarrow$ finite temperature $T = 1/L_t$

Lattice Dirac operator

- Partition function (integrating out quarks):

$$Z = \int \mathscr{D}\psi\,\mathscr{D}\bar{\psi}\,\mathscr{D}U\ \mathrm{e}^{-S_g[U]\ -\ \bar{\psi}\{D[U]+M\}\psi}$$

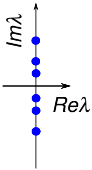
$$= \int \mathscr{D}U\ det\{D[U]+M\}\cdot \mathrm{e}^{-S_g[U]}$$

- Statistical physics system (4-dimensional)
- Dynamical variables: $U_i \in SU(N_c)$ on lattice links

Dirac operator: $D[U]$

- Discretized differential operator depending on U-s
- $\propto V \times V$ sparse matrix (V–volume)
- $(D[U]+M)^{-1}$ appears in physical quantities
- Small eigenvalues (eigenvectors) physically important

The structure of the Dirac operator



Symmetries of D

$\rightarrow$ Spectrum imaginary, symmetric around 0

- probability distribution of $[U]$ $\Rightarrow$ random $D[U]$ with given distribution
- distribution of $D[U]$ $\Rightarrow$ physical quantities
- eigenvalue ststistics of $D[U]$ $\Rightarrow$ bulk termodynamics

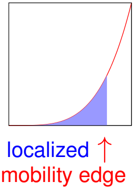
What do we know about the spectral statistics of $D[U]$?

Is the detailed dynamics important or it is already given by the symmetries?

Spectrum of sparse random matrices

Anderson model

- one-electron Hamiltonian in disorderer crystal
  - random on-site “energies”
  - constant hopping terms to nearest neighbor sites
  - all other matrix elements zero
- localized states at the band edge
- delocalized states at the band center

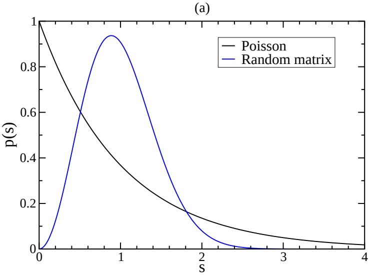


- mobility edge controlled by disorder

Localization and spectral statistics

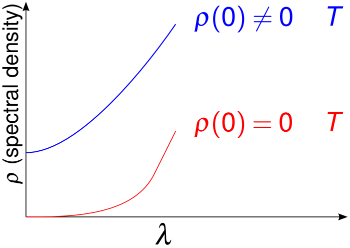
Level spacing distribution

- Level spacing: $\lambda_{n+1} - \lambda_n$
- Unfolding: $s = \frac{\lambda_{n+1} - \lambda_n}{\langle \lambda_{n+1} - \lambda_n \rangle}$
- Two extremes:
  - λ_n statistically independent (Poisson) $\Rightarrow$ $p(s) = \exp(-s)$
  - Random matrix statistics



How about the Dirac operator?

- $\lambda = 0$ special point (symmetry).
- Transition at $T_c \approx 200\text{MeV}$:



$\rho(0) \neq 0$ $T < T_c$ hadronic phase

$\rho(0) = 0$ $T > T_c$ quark-gluon plasma

$\rho(0) \neq 0 \Rightarrow$ statistics of low eigenvalues of $D[U]$ described by random matrix theory
analytically (σ-model) $+$ numerically (lattice QCD)

Eigenvalue statistics $T > T_c$ (quark-gluon plasma)

in different regions of the spectrum

unfolded level spacing distribution $s = \frac{\lambda_{n+1} - \lambda_n}{\langle \lambda_{n+1} - \lambda_n \rangle}$

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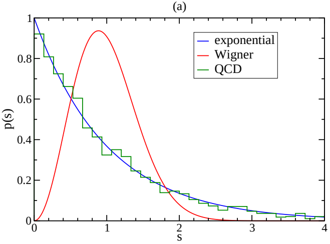

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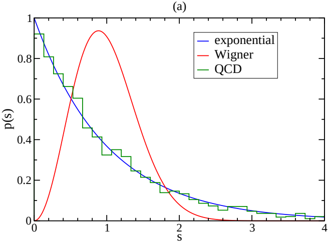

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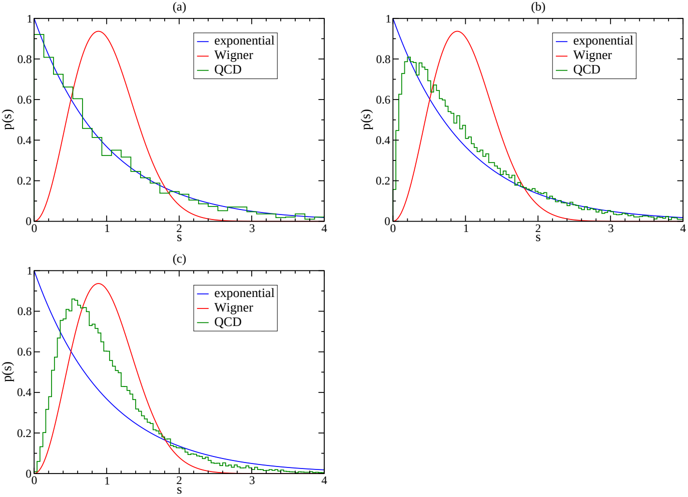

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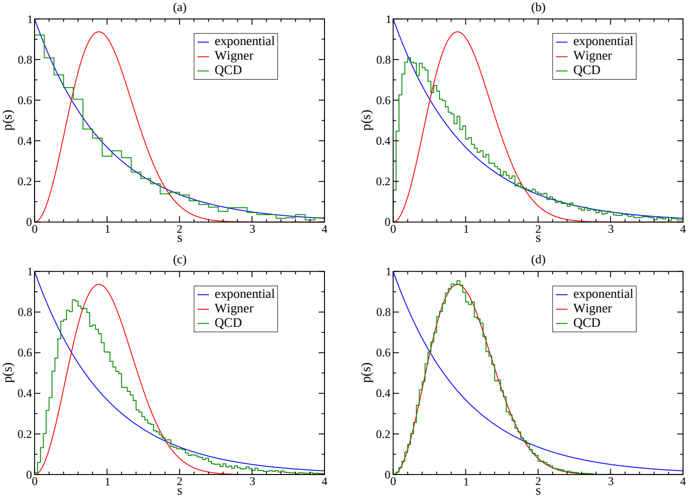
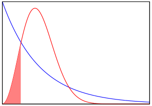
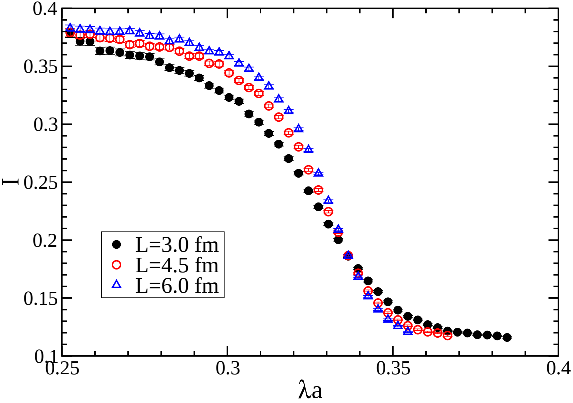


Integrated level spacing distribution

$$I = \int_0^{0.5} p(s)ds$$





Finite size scaling

verify critical behavior and compute ν

- Parameters:
  - λ (location in the spectrum) relevant
  - μ leading irrelevant
- $I(\lambda, \mu, L)$ dimensionless, RG invariant
- $I(\lambda, \mu, L) = I(b^{1/\nu}\lambda,\ b^{y_\mu}\mu,\ b^{-1}L)$
- Only one relevant variable
  - Close to the critical point: $b^{y_\mu}\mu \approx 0$
  - Block all systems to "standard size" $\rightarrow$ $b \propto L$
- Scaling function:

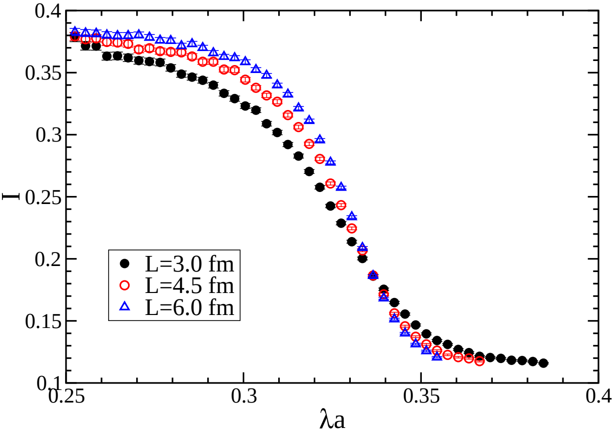
$$I(\lambda, \mu, L) = f\left(L^{1/\nu}(\lambda - \lambda_c)\right)$$

- For finite L $I(\lambda)$ analytic $\rightarrow$ f analytic

Finite size scaling

$$I(\lambda, \mu, L) = f\left(L^{1/\nu}(\lambda - \lambda_c)\right)$$

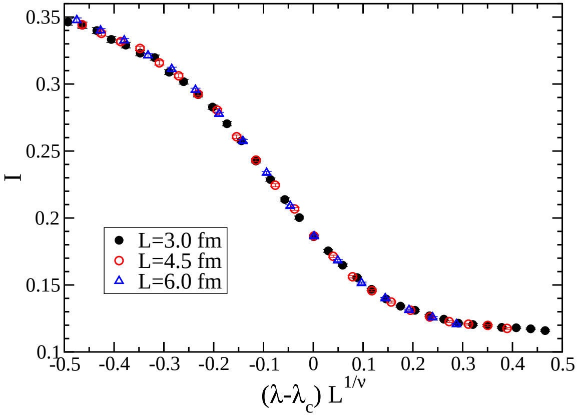
Is it possible to choose ν and λ_c to have data collapse?



Finite size scaling

$$I(\lambda,\mu,L) = f\left(L^{1/\nu}(\lambda - \lambda_c)\right)$$

Yes! fit polynomial to $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots$ and λ_c, ν



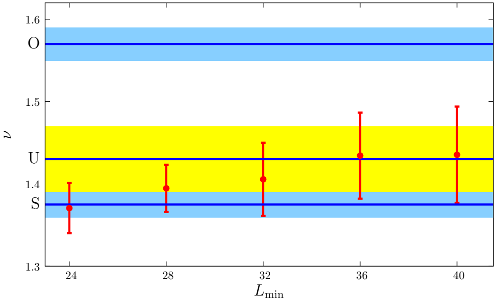
Corrections to scaling

- Use only systems larger than $L_{\min}$ for the fit
  - system sizes: $L^3 = 24^3, 28^3, 32^3, 36^3, 40^3, 48^3, 56^3$
  - $56^3 \times 4$ $4 \cdot 10^6$ dimensional

K. Slevin & T. Ohtsuki, PRL 82 (1999)

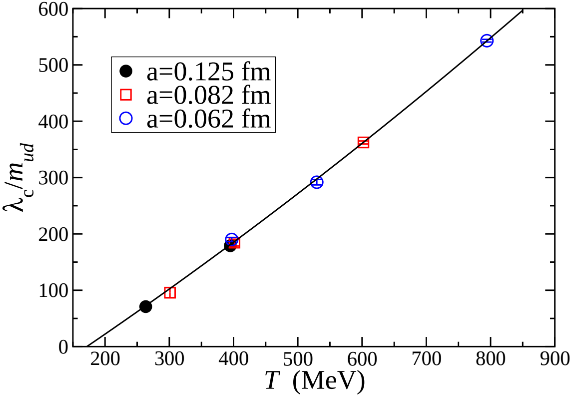
K. Slevin & T. Ohtsuki, PRL 78 (1997)

Y. Asada, K. Slevin & T. Ohtsuki, J.Phys.Soc.Jpn. 74 (2005)



- ν compatible with unitary Anderson model

The temperature controls the mobility edge



for fixed lattice size the system gets more ordered! $\longrightarrow$

Conclusions

- Phase transition or not?
  - The transition to QGP is only a cross-over
  - Have we found a genuine phase transition?
  - No! In QCD no thermodynamic quantity is singular
  - Genuine transition in QCD-like models?
- Relevance of localized Dirac modes in QGP
  - Mobility edge $\rightarrow$ effective gap (like large m_q)
  - Screening masses
  - External magnetic field and QCD thermodynamics
- References:
  - TGK, PRL 104 (2010); TGK, F. Pittler, PRL 105 (2010);
  - F. Bruckmann, TGK, S. Schierenberg, PRD 84 (2011);
  - TGK, F. Pittler, PRD 86 (2012);
  - M. Giordano, TGK, F. Pittler, PRL 112 (2014)