Dynamics of the short-term memory

Full depletion model

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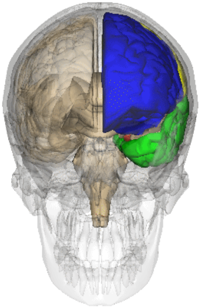
Overview of the talk

- working memory
- short-term plasticity
- full depletion model
- analytic approximation
- PyDSTool package
- dynamical system view:
    - limit cycle classes
    - bifurcation diagrams
- generalized Taken-Bogdanov system
- discussion

The human brain

Numbers:

- weighs on average about 1.5 kg
- volume of around 1130 cm^3 in women and 1260 cm^3
- composed of neurons, glial cells, and blood vessels
- $2 \cdot 10^{11}$ neurons $\sim$ nr. of stars in the Milky Way
- $1.25 \cdot 10^{14}$ synapses in the cerebral cortex alone



Micheva et al., Neuron, 2010

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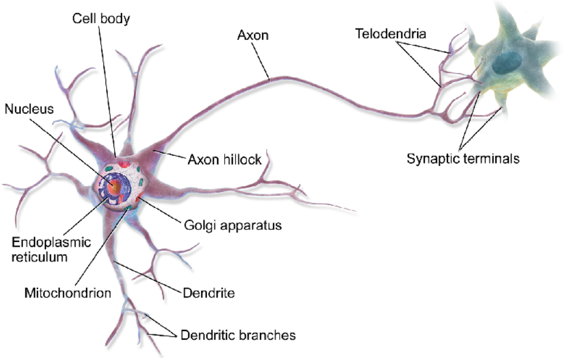
Neurons

- neuron - fundamental structural functional unit of the brain
- discrete cells and not continuous with other cells
- information flows from the dendrites to the axon via the cell body



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Synapse

Two different types of synapses:

- chemical synapse: electrical activity is converted into the release of a neurotransmitter
- electrical synapse: cell membranes are connected by channels that are capable of passing electric current

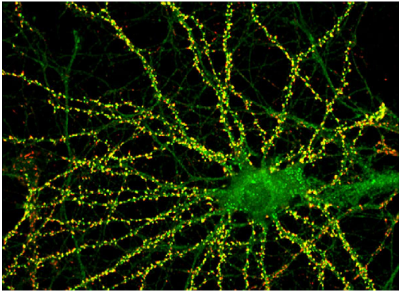

Image Credit: Kennedy lab, Caltech

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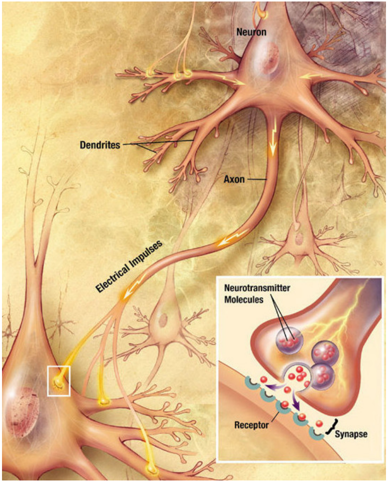
Chemical synapse

Synaptic transmission:

- activation of voltage-gated calcium channels
- release of neurotransmitters
- neurotransmitter receptors
- opening ion channels - postsynaptic potential and/or chemical messengers

Types (neurotransmitter):

- glutamatergic (excitatory)
- GABAergic (inhibitory)
- ...



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Synaptic plasticity and memory

Plasticity: synapses can strengthen or weaken over time

- long-term plasticity - from minutes to hours
  - long-term potentiation (LTP)
  - long-term depression (LTD)
- short-term or transient synaptic plasticity - from milliseconds to a few minutes

Memory: information encoding, storage and retrieval

- sensory memory - less than one second, limited capacity
- short-term memory - several seconds to minutes (without rehearsal), limited capacity
  - acoustic code + visual code
- long-term memory - sometimes a whole life span, immeasurably large capacity
  - encodes information semantically

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Test

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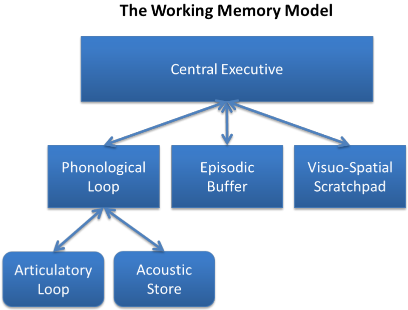
Working memory model

- holding and manipulating multiple information in the mind
  - “magical number seven“ (Miller, 1956)
  - link between working memory and learning and attention
- Baddeley’s model (Baddeley and Hitch, 1974):



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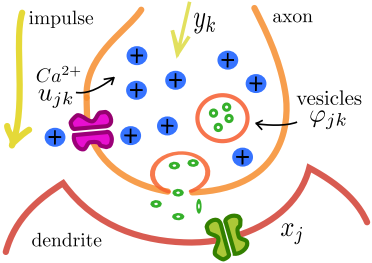
Short-term synaptic plasticity

- Tsodyks-Markram model (5 parameters)
  - the reservoir $\varphi(t)$ of vesicles with neurotransmitters
  - the Ca-concentration $u(t)$ influencing the release probability of vesicles

$$\dot{u} = \frac{1-u}{T_u} + \alpha(U-u)y$$

$$\dot{\varphi} = \frac{1-\varphi}{T_\varphi} - \beta\varphi u y$$



- $y \in [0,1]$ is the pre-synaptic activity level (the firing rate of the other neuron)

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Synaptic theory of working memory

- short-term synaptic plasticity and selective neural populations (Mongillo et al., Science, 2008)

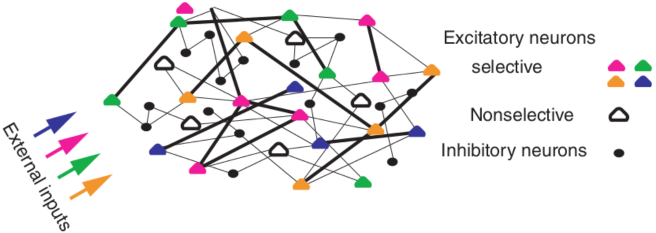


Image Source: Mongillo et al., Science, 2008

Synaptic theory of working memory

- loading in and reading out memory items (Mongillo et al., Science, 2008)

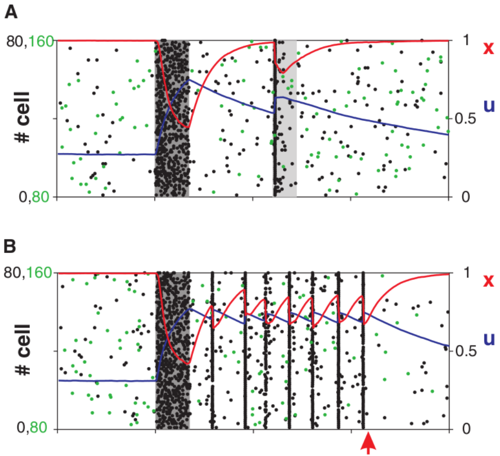


Image Source: Mongillo et al., Science, 2008

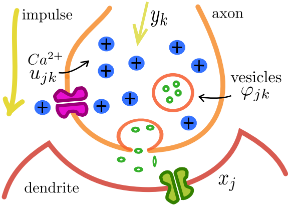
Full depletion model

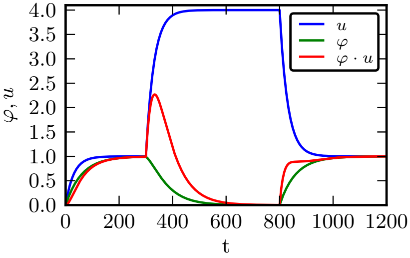
- for the stricter control of the dynamics (3 parameters)

$$\dot{u} = \frac{U_y - u}{T_u} \qquad U_y = 1 + (U_{max} - 1)y$$

$$\dot{\varphi} = \frac{\Phi_u - \varphi}{T_\varphi} \qquad \Phi_u = 1 - \frac{uy}{U_{max}}$$

- full depletion after sustained pre-synaptic firing:
  - $y(t) = 1$ for $t \in [300, 800]$
  - $T_\varphi = 60$, $T_u = 30$, $U_{max} = 4$



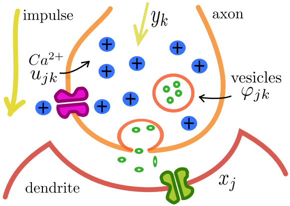


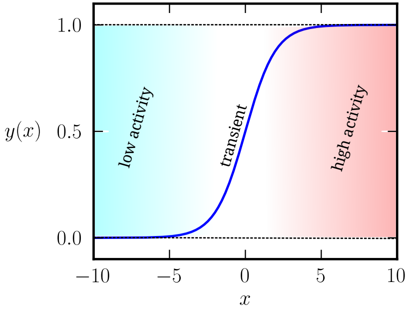
Neural activity

- dynamics of the neural activity:

$$\dot{x}_j = -\Gamma x_j + \sum_k (w_{jk} + z_{jk} u_{jk} \varphi_{jk}) y_k$$

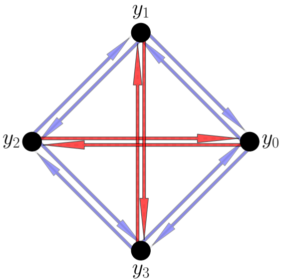
$$y_k = y(x_k) = \frac{1}{1 + e^{a(b - x_k)}}$$

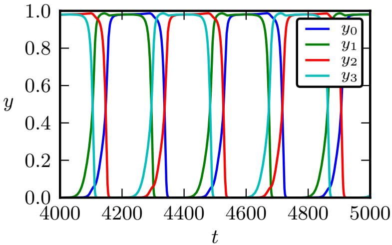




Clique encoding recurrent network

- nearest-neighbours - excitatory weights w_0 (blue arrows)
- others - inhibitory weights z_0 (red arrows)





- $T_\varphi = 60$ are $T_u = 30$ are kept fixed

Symmetric approximation

- assume that the initial conditions are the same for all neurons

$$\begin{cases} u_i(t) = u_j(t), \\ \varphi_i(t) = \varphi_j(t), \\ x_i(t) = x_j(t), \qquad \forall i, j = \overline{1, N} \end{cases} \tag{1}$$

- dimension reduction:

$$\dot{x} = -\Gamma x + [2w_0 + (N-3)z_0 u\varphi]y. \tag{2}$$

- fixpoints of the reduced system:

$$\begin{cases} \dot{x} = 0, & \Rightarrow \quad \Gamma x = [2w_0 + (N-3)z_0 u\varphi]y \\ \dot{u} = 0, & \Rightarrow \quad u = 1 + (U_{max} - 1)y \\ \dot{\varphi} = 0, & \Rightarrow \quad \varphi = 1 - \frac{uy}{U_{max}} \end{cases} \tag{3}$$

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Dynamical systems view: fixpoints

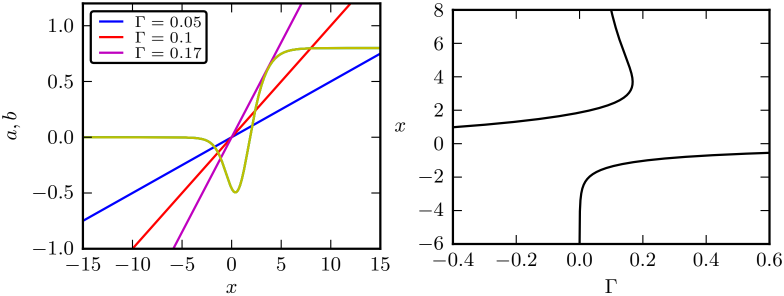
- substituting the u and φ variables

$$\Gamma x = y(x) \cdot \left\{ 2w_0 + (N-3) z_0 \overbrace{\left[1 + (U_{max} - 1) y(x) \right]}^{u} \cdot \right.$$
$$\left. \underbrace{\left[1 - \frac{y(x) + (U_{max} - 1) y^2(x)}{U_{max}} \right]}_{\varphi} \right\}$$

- solutions can be found grafically
- note that the left hand side is always a straight line

Dynamical systems view: Γ parameter

- fixpoints and bifurcation diagram
- parameters: $w_0 = 0.4$, $z_0 = -1$, $U_{max} = 4$



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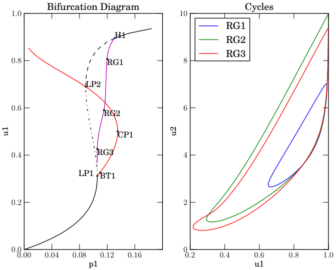
PyDSTool package

- supports:
    - symbolic math
    - optimization
    - phase plane analysis
    - continuation and bifurcation analysis
    - data analysis, etc.



Left: Bifurcation diagam.
Right: Limit cycles.

Clewley RH, Sherwood WE, LaMar MD, Guckenheimer JM (2007) PyDSTool, a software environment for dynamical systems modeling. URL http://pydstool.sourceforge.net

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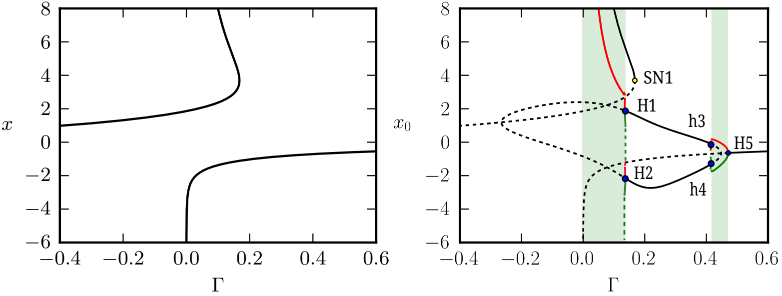
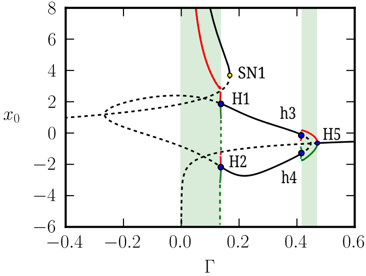
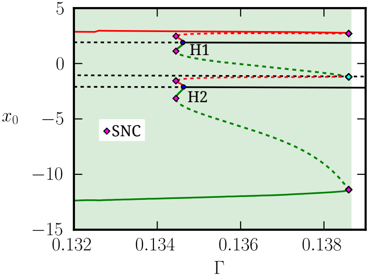
Dynamical systems view: Γ parameter

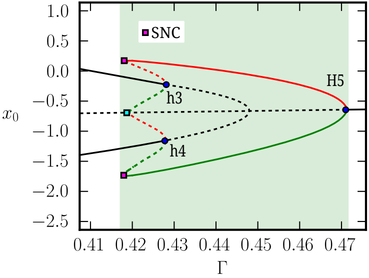
- full bifurcation diagram



Dynamical systems view: Γ parameter

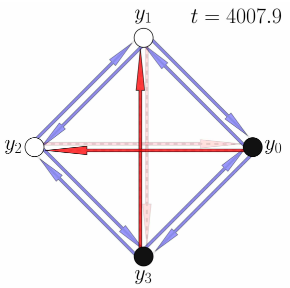


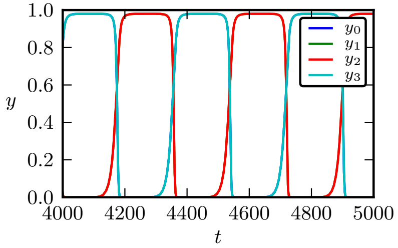




Dynamical systems view:
Γ parameter - limit cycles (class I.)

- synchronised dynamics





Neural activity.
Parameters: $\Gamma = 0.1$,
$w_0 = 0.4$, $z_0 = -1$, $U_{max} = 4$.

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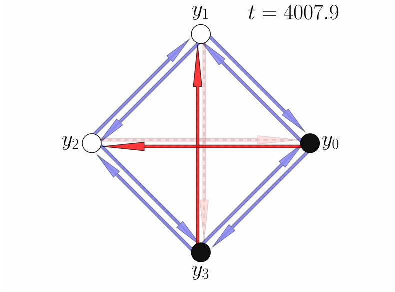
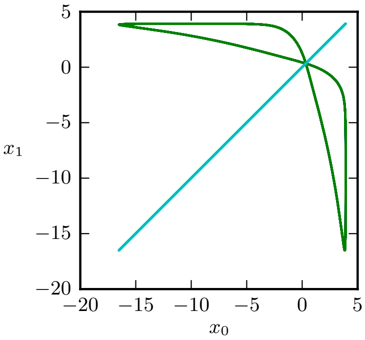
Dynamical systems view: Γ parameter - limit cycles (class I.)

- synchronised dynamics

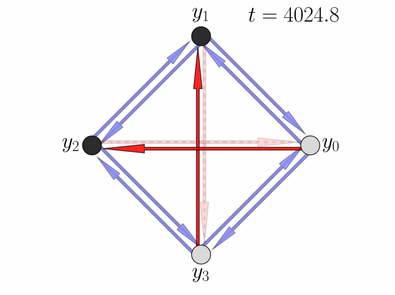


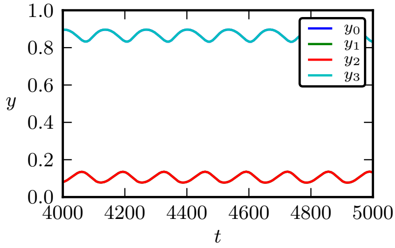


Limit cycles.
Parameters: $\Gamma = 0.1$,
$w_0 = 0.4$, $z_0 = -1$, $U_{max} = 4$.

Dynamical systems view: Γ parameter - limit cycles (class IV.)

- small amplitude synchronised dynamics





Neural activity.
Parameters: $\Gamma = 0.1345$,
$w_0 = 0.4$, $z_0 = -1$, $U_{max} = 4$.

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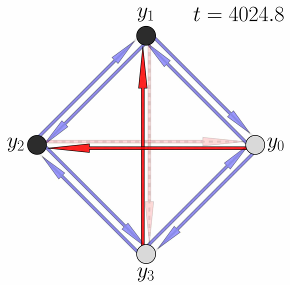
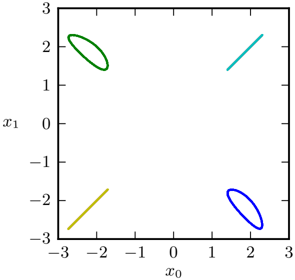
Dynamical systems view: Γ parameter - limit cycles (class IV.)

- small amplitude synchronised dynamics





Limit cycles.
Parameters: $\Gamma = 0.1345$, $w_0 = 0.4$, $z_0 = -1$, $U_{max} = 4$.

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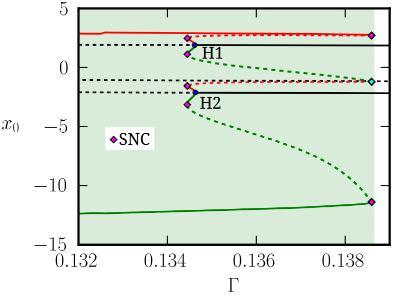
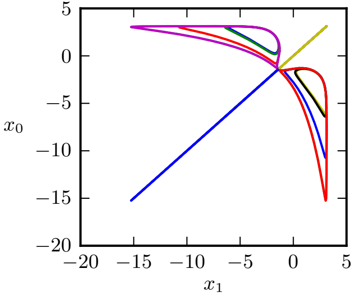
Bifurcations with limit cycles

- small limit cycles appear by Hopf-bifurcation (H)
- saddle-node bifurcation of limit cycles (SNC)
- limit cycles merge by homoclinic bifurcation





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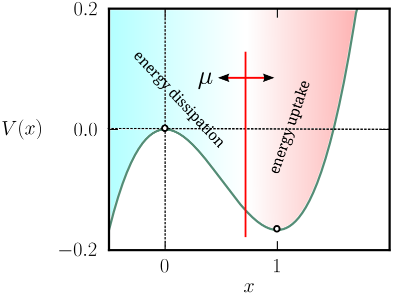
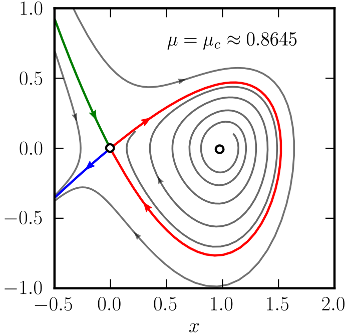
Taken-Bogdanov system

- example for Homoclinic bifurcation
- mechanical system with a potential $V(x)$ and adaptive velovity-dependent forces:

$$\ddot{x} = (x-\mu)\dot{x} - V'(x) \qquad \dot{x} = y$$

$$V(x) = x^3/3 - x^2/2 \qquad \dot{y} = (x-\mu)y + x(1-x)$$





Generalized Taken-Bogdanov systems

- dissipation changes to anti-dissipation when $\mu = V(\mathbf{x})$:

$$\ddot{\mathbf{x}} = \alpha(\mu - V(\mathbf{x}))\dot{\mathbf{x}} - \nabla(\mathbf{x})$$

- fixpoints at the minima and maxima of $V(\mathbf{x})$:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{y} & \mathbf{y}^* &= 0 \\ \dot{\mathbf{y}} &= \alpha(\mu - V(\mathbf{x}))\mathbf{y} - \nabla V(\mathbf{x}) & \nabla V(\mathbf{x}^*) &= 0 \end{aligned}$$

- protential with predefined minima $V(\mathbf{x_n^*}) \approx V_n$, $n = \overline{1, M}$:

$$V(\mathbf{x}) = \prod_n \left(g_n(\mathbf{x} - \mathbf{x_n^*}) + \frac{V_n}{p_n} \right)$$

$$g_n(\mathbf{z}) = \tanh(\mathbf{z}^2 / {z_n}^2), \qquad p_n = \prod_{j \neq n} \left(g_n(\mathbf{x_n} - \mathbf{x_j^*}) + \frac{V_j}{p_j} \right)$$

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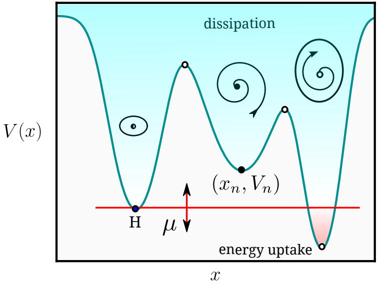
Generalized Taken-Bogdanov systems

- 2 dimensional example

$$\dot{x} = y \qquad\qquad y^* = 0$$
$$\dot{y} = \alpha(\mu - V(x))y - V'(x) \qquad\qquad V'(x^*) = 0$$

- stability of fixpoints:

$$J = \begin{pmatrix} 0 & 1 \\ V''(x^*) & \alpha(\mu - V(x^*)) \end{pmatrix}$$



$$d = \det(J) \quad = V''(x^*) \Rightarrow \text{saddles}$$
$$t = \mathrm{Tr}(J) \quad = \alpha(\mu - V(x^*))$$

$$\lambda_{1,2} = \frac{t \pm \sqrt{t^2 - 4d}}{2} \Rightarrow$$

when $\mu \to V(x^*) \Rightarrow t \to 0 \Rightarrow$ series of Hopf-bifurcations at x_n^*

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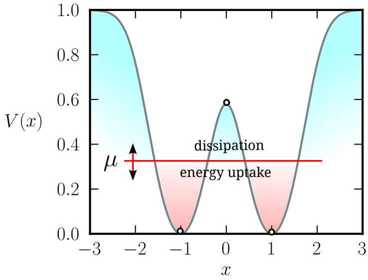
Double potential well

- system with a symmetric double potential well:

$$\ddot{x} = \alpha(\mu - V(x))\dot{x} - V'(x)$$

$$V(x) = \left(g_1(x - x_1^*) + \frac{V_1}{p_1}\right) \cdot \left(g_2(x - x_2^*) + \frac{V_2}{p_2}\right)$$



$$x_1 = -1$$
$$x_2 = +1$$
$$V_1 = V_2 = 0$$
$$z_1 = z_2 = 1$$
$$p_1 = p_2 = 1$$

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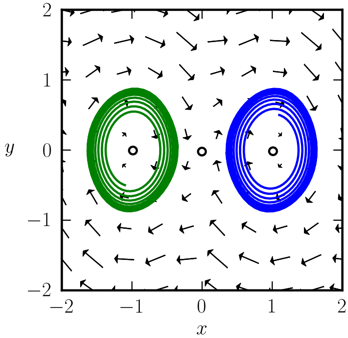
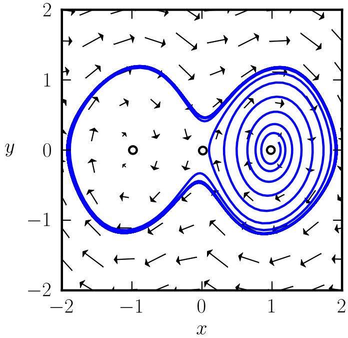
Merging limit cycles





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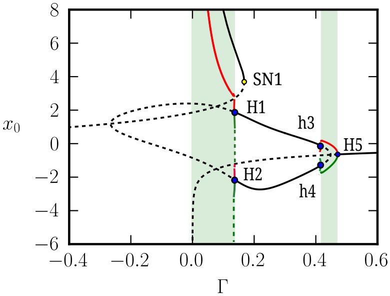
Possible interpretations

- full bifurcation diagram



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- phonological loop + phonological store
  - 3Xgh-3Xgh-3Xgh...
  - 3Xgh-1579-3Xgh-1579-3Xgh-1579...

Summary

- memory and synaptic plasticity
- working memory
- new model for transient synaptic plasticity
- implemented in a recurrent neural network
- dyinamical systems type study
- analytic method for fixpoint analysis
  - parameter regions
  - limit cycles classes
- merging limit cycles
- new example system
- interpretations

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