Area law violation for the mutual information in a nonequilibrium steady state

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Introduction: Area laws

- Area laws often emerge for correlations in many-body states
- Historical origin: black hole physics, holographic principle
- Entanglement in *ground states* of local Hamiltonians
- Crucial role in classical simulation (DMRG, MPS, PEPS)
- Exception: quantum critical systems, logarithmic violations
- Best understood in 1D with CFT
- Ideal testing ground: free fermion systems
- Area-law violation persists in higher dimensions

Introduction: Mutual information

- Measures *total* correlations

$$I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

- Strict area-law for Gibbs-states, for any *non-zero* temperature and *local* Hamiltonian! Wolf et.al. 2008
- CFT prediction for 1D free fermions (c=1)

$$I(A:B) \approx \frac{c}{3} \log\left(\frac{\beta}{\pi a}\right)$$

Calabrese & Cardy 2004

- Higher dimensions

$$I(A:B) = O(M^{D-1} \log(\beta))$$

Bernigau et.al. 2013

- What happens *out of equilibrium*?

The model

- Initial state:

$$\rho_0 = \frac{1}{Z_\ell} \mathrm{e}^{-\beta_\ell \mathcal{H}_\ell} \otimes \frac{1}{Z_r} \mathrm{e}^{-\beta_r \mathcal{H}_r}$$

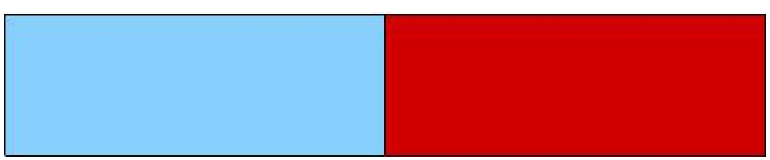
$$\mathcal{H}_\ell = -\frac{1}{2} \sum_{m=-\infty}^{-1} \left(c_m^\dagger c_{m+1} + c_{m+1}^\dagger c_m \right)$$

$$\mathcal{H}_r = -\frac{1}{2} \sum_{m=1}^{\infty} \left(c_m^\dagger c_{m+1} + c_{m+1}^\dagger c_m \right)$$

- Time evolution:

$$\rho_t = \mathrm{e}^{-i\mathcal{H}t} \rho_0 \mathrm{e}^{i\mathcal{H}t}$$

$$\mathcal{H} = -\frac{1}{2} \sum_{m=-\infty}^{\infty} \left(c_m^\dagger c_{m+1} + c_{m+1}^\dagger c_m \right)$$



t = 0

t > 0

The steady state is a GGE

- Defined through local observables

$$\lim_{t\to\infty} \mathrm{Tr}\,(\rho_t \mathcal{O}_S) = \mathrm{Tr}\,(\rho_\infty \mathcal{O}_S)$$

Ho & Araki 2000
Ogata 2002
Aschbacher & Pillet 2002

- NESS is a generalized Gibbs ensemble (GGE)

$$\rho_\infty = \frac{1}{Z} \mathrm{e}^{-\beta \mathcal{H}_{\mathrm{eff}}}, \quad \mathcal{H}_{\mathrm{eff}} = \sum_{n=0}^{\infty} \left(\mu_n^+ Q_n^+ + \mu_n^- Q_n^- \right) \qquad \beta = (\beta_\ell + \beta_r)/2$$

- Conserved charges and associated "temperatures"

$$Q_n^+ = -\frac{1}{2} \sum_{m=-\infty}^{\infty} \left(c_m^\dagger c_{m+n} + c_{m+n}^\dagger c_m \right)$$

$$Q_n^- = -\frac{i}{2} \sum_{m=-\infty}^{\infty} \left(c_m^\dagger c_{m+n} - c_{m+n}^\dagger c_m \right)$$

$$\mu_n^+ = \delta_{n,1}\,, \quad \mu_n^- = \begin{cases} \frac{4}{\pi} \frac{\beta_\ell - \beta_r}{\beta_\ell + \beta_r} \frac{n}{n^2 - 1} & n \text{ even,} \\ 0 & n \text{ odd.} \end{cases}$$

$\mathcal{H}_{\mathrm{eff}}$ is long range!

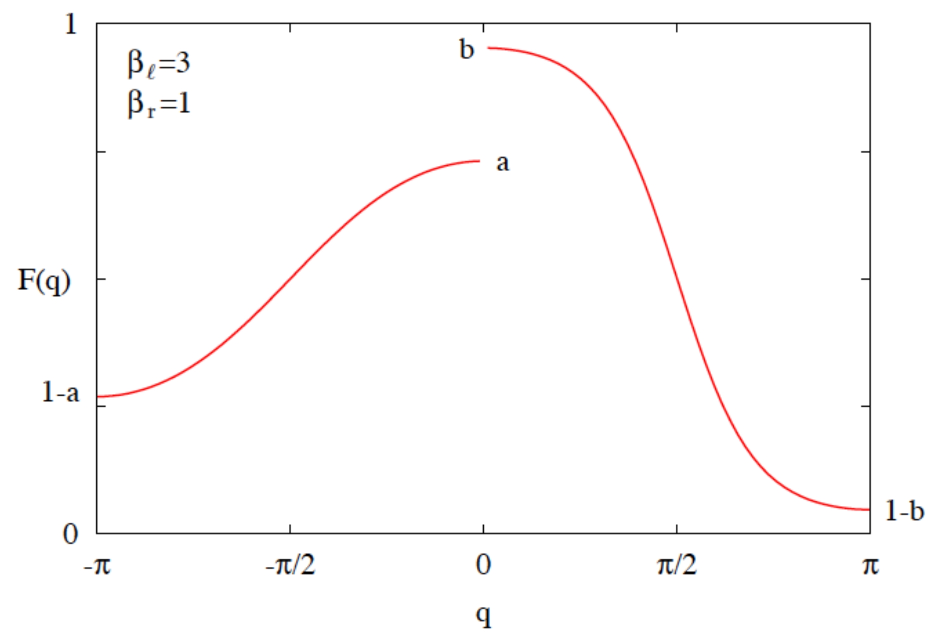
Steady state correlations

- Fermionic correlation matrix:

$$C_{mn} = \mathrm{Tr}\left(\rho_\infty c_m^\dagger c_n\right) = \int_{-\pi}^{\pi} \frac{\mathrm{d}q}{2\pi} \mathrm{e}^{iq(m-n)} F(q)$$

- Toeplitz matrix with symbol

$$F(q) = \begin{cases} \frac{1}{\mathrm{e}^{\beta_r \omega_q}+1} & q \in (-\pi, 0) \\ \frac{1}{\mathrm{e}^{\beta_\ell \omega_q}+1} & q \in (0, \pi) \end{cases}$$



- Simple physical interpretation!

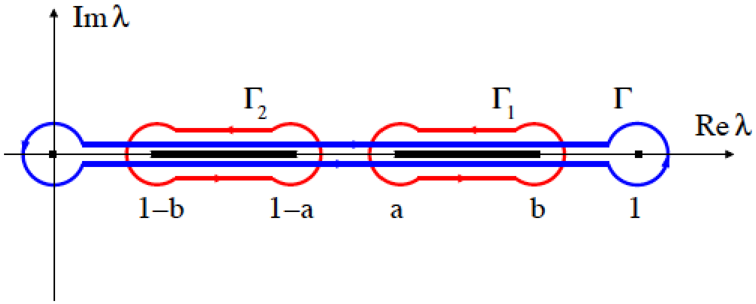
Steady-state mutual information

- Calculate entropy from correlation matrix eigenvalues

$$S_L = \sum_{k=1}^{L} s(\lambda_k) \qquad s(\lambda) = -\lambda \ln \lambda - (1-\lambda)\ln(1-\lambda)$$

- Integral representation (Jin & Korepin 2006)

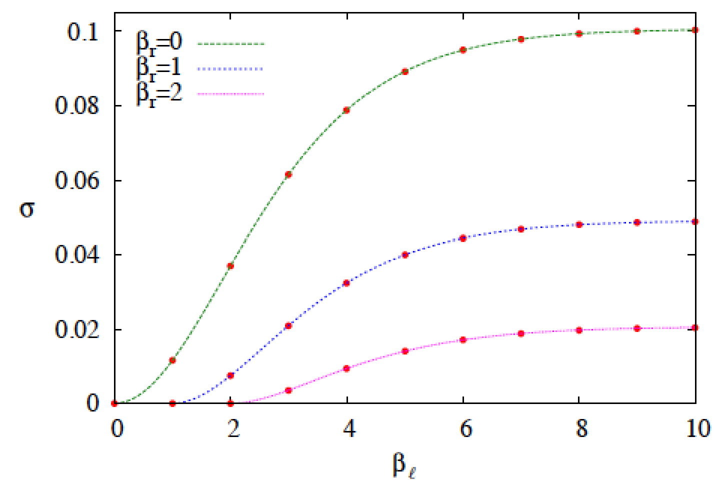
$$S_L = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}\lambda\, s(\lambda)\, \frac{\mathrm{d}\ln D_L(\lambda)}{\mathrm{d}\lambda}$$



- Use Fisher-Hartwig formula
- Result:

$$I(A:B) = \sigma \ln L + \text{const.}$$

$$\sigma = \frac{1}{\pi^2}\left[a\,\mathrm{Li}_2\left(\frac{a-b}{a}\right) + (1-a)\mathrm{Li}_2\left(\frac{b-a}{1-a}\right) + b\,\mathrm{Li}_2\left(\frac{b-a}{b}\right) + (1-b)\mathrm{Li}_2\left(\frac{a-b}{1-b}\right)\right]$$

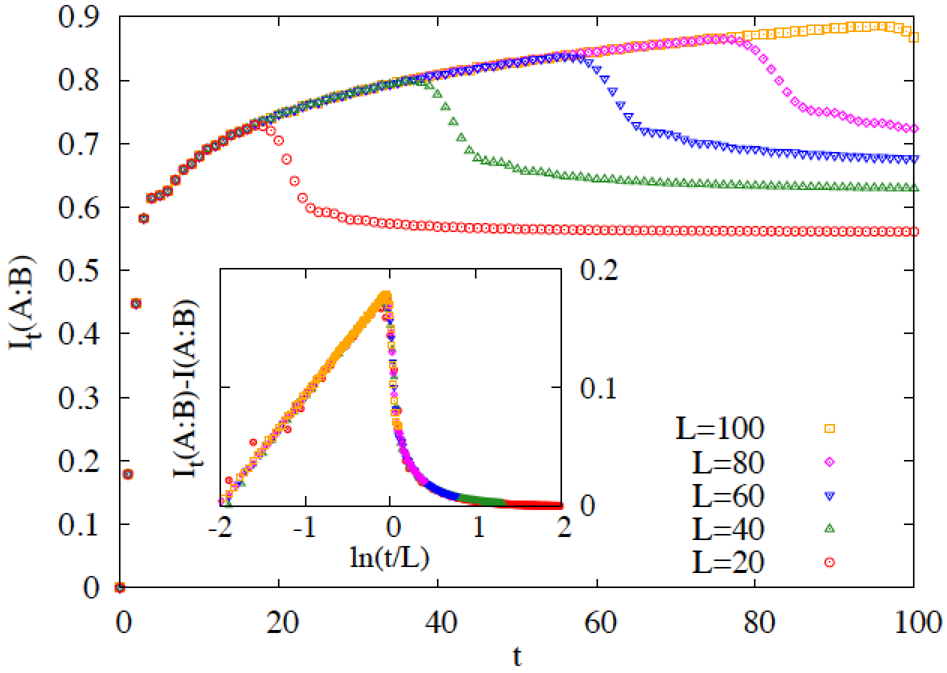


Dynamics of mutual information

- Time dependent correlations:

$$C_{mn}(t) = i^{n-m} \sum_{k,l \in \mathbb{Z}} i^{k-l} J_{m-k}(t) J_{n-l}(t) C_{kl}(0)$$

- Logarithmic growth until front leaves subsystem
- Sharp drop and convergence to steady-state value
- Reminiscent to local quench scenario



Conclusions & Outlook

- Area-law violation outside zero temperature!
- Long-range GGE is necessary but not sufficient!
- Example: magnetic field quench in TI chain
- Violation due to time reversal symmetry breaking?
- Further examples for thermally driven spin chains?
- Long-range spin correlations necessary?
- Nonequlibrium CFT calculation?
- Universality?