Extreme Statistics and Physical Applications – special course

Homeworks from the part of the theory of extreme statistics.

1. (*10 %*) Calculate the linear corrections to a_p and b_p in the RG transformation of an eigenfunction (that is, $a_p^{(1)}$ and $b_p^{(1)}$ paremetrized by γ and γ').

2. (*15 %*) Show that $M_{\gamma+\Delta\gamma}(x)$ is to linear order $M_\gamma\left(x+\Delta\gamma\,\psi_{\gamma,\gamma'=0}(x)\right)$, i. e., the $\gamma'=0$ eigenfuncion can be calculated by differentiation of distribution functions of the fixed line in terms of the parameter γ.

3. (*12 %*) Give a limit formula for γ' in terms of $g(z)$ by differentiating $g(a_N x+b_N)-g(b_N)\approx f(x)+\epsilon_N f'(x)\,\psi(x)$ knowing that $\psi'''(0)=\gamma-\gamma'$.

4. (*12 %*) Give a limit formula for γ' in terms of $g(z)$ using $d\ln|\epsilon_N|/d\ln N\to\gamma'$, and show its equivalence to the expression from homework No. 3.

5. (*8 %*) Assume for the parent distribution the asymptote $\mu(z)\approx 1-Az^{-\nu}(\ln z)^\eta$ and determine γ'.

6. (*8 %*) Same as No. 5 with $\mu(z)\approx 1-Az^\theta e^{-Bz^\delta}$.

7. (*8 %*) Calculate γ' for the exponential parent $\mu(z)=1-e^{-z}$.

8. (*30 %*) Renormalization to second order for $\gamma=\gamma'=0$: Since the eigenvalue is unity, a nonlinear correction should be taken into account. Assume that $\epsilon'=\epsilon-r\epsilon^2$, take the trial function
$$M(x,\epsilon)\approx M\left(x-\frac{\epsilon}{2}x^2+\epsilon^2\psi_2(x)\right),$$
then calculate $\psi_2(x)$.

9. (*15 %*) Central limit theorem: Calculate the leading finite size correction to the Gaussian limit distribution, assuming the existence of a finite third moment.

10. (*20 %*) Central limit theorem: Discuss the finite size correction to the Lévy distribution.

11. a) (*25 %*) Simulate the extreme value statistics of two parent distributions belonging to the FTG domain of attraction, and show that the previously found $\psi(x)$ finite size correction is indeed appropriate.
    b) (*20 %*) Use the standardization $\langle x\rangle=0$, $\langle x^2\rangle=1$ on the simulation data. What is the theoretical correction function to which they should fit?

12. (*20 %*) Show that, for large k, the limit probability density function
$$P^{(k)}(x)=\frac{1}{(k-1)!}f'(x)\exp\left\{-kf(x)-\exp\left[-f(x)\right]\right\}$$
of the kth maximum converges to a Gaussian, if scaled appropriately.

13. (*20 %*) Plot the limit density of the first gap, $P_G(\Delta)$, for $\gamma=-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{3}{2}$.

14. (*30 %*) Show that the integrated limit distribution of the kth gap $\Delta=(z_k-z_{k+1})/a_N$ is
$$M_G^{(k)}(\Delta)=1-\frac{1}{k!}\left\langle e^{-kf(x+\Delta)}\right\rangle,$$
and the average is $\langle\Delta\rangle^{(k)}=\Gamma(k-\gamma)/k!$. Give the distribution function explicitly for $\gamma=0,-1$.

15. (*10 %*) Show that the limit distribution of maxima from the idd variables, obeying $\mu_i(z) = 1 - 1/(i+z)$, cannot be transformed by a linear change of variable to any of the iid fixed point distributions.